Identifying the Learning Difficulties of Freshmen in Mathematics Teacher Training Department in Function Graphs and Derivatives

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Abstract

The present study investigated the learning difficulties of the freshmen in the department of mathematics teacher training in the concept of derivatives and drawing rational function graphs and their possible causes. The study used the relational survey model. The study was conducted during the first semester of the academic year 2009-2010 with 84 students taking the course Analysis I. The sample contains 45 male and 39 female students who took the course Analysis I. In this study used five questions. The questions' Cronbach's alpha reliability coefficient was found to be 0,79. In the light of the results obtained, various suggestions are made so as to minimize learning difficulties in students and pre-service teachers and to achieve meaningful learning.

Keywords: Learning difficulties, derivative, pre-service teachers, rational function graphs

Introduction

Instruction mainly aims to achieve learning. As the stakeholders of instruction, students and teachers play various roles for ideal learning. Each course contains basic principles to be implemented so as to ensure ideal learning. (3) argues that the basic principles in mathematics teaching include attaching importance to laying conceptual foundations, prerequisite relationships, key concepts, and research and developing positive attitudes toward mathematics (pp. 10-13). Lack of regard for these principles is among the possible causes of the learning difficulties experienced by students. For (12), many individuals studying mathematics ay any level may have serious difficulties even in very simple concepts. The literature has noted that the origins of learning difficulties are the way knowledge is constructed in the mind (10); the inability to interpret verbal problems, inadequate algebra instruction, insufficient levels of readiness among students, lack of conceptual understanding (15), abstract concepts (5, 9), inadequate understanding of basic concepts (14), and lack of motivation (9).

Mathematics subjects are intertwined like the rings of a chain. Any subject is linked to those that come before and after it. Therefore, the links between a subject and others should be highlighted. For as (8) argue, it is difficult for students with difficulties in any subject to achieve success in subsequent subjects. The subject of derivatives is one of the best examples of this fact. (4) claims that the subject of derivatives is used to explain the concepts of tangent slope, velocity and acceleration in physics, reaction rate in chemistry, marginal income and marginal price in economics (4, p. 113). Understanding and making sense of the concept of derivatives requires subject and conceptual knowledge of many important mathematical subjects and their relations with each other. To give an example, making sense of derivatives requires knowledge of geometry and certain mathematical concepts such as functions, limit, tangent, slope, continuity and rate of change (6, p.224). (18) maintains that continuity, limit, functions and relations are the primary factors that influence the teaching of derivatives. A lack of understanding in any of these concepts will lead to difficulties for students in understanding the subject of derivatives. As a matter of fact, (6) identified learning difficulties and misconceptions in students about derivative-tangent, derivative-limit, and derivative-rate of change relationships.

(17), on the other hand, found that students had misconceptions about derivative at a certain point and the derivative function. (12) found that students acquired the ability to obtain the derivative of a mathematical statement as a rule, rather than learning what derivative means. Moreover, no significant difference was found between female and male students in implementing cognitive behaviors. (13) found that the concept of derivatives was not learned meaningfully. Although the students in the sample group were skilled in derivatives, they tried to memorize the formulae and use them by rote. (2) demonstrated that computer-assisted constructivist learning environments contribute more to the construction and learning of the concept of derivatives when compared to constructivist learning environments. The students in the experiment group were more successful in conceptual understanding than the control group. No significant difference was found between the two groups in questions concerning operational knowledge and problem solving.

The present study investigated the learning difficulties of the freshmen in the department of mathematics teacher training in the concept of derivatives and drawing rational function graphs and their possible causes. The subject of derivatives was chosen since students are confronted with it in different branches of science (physics, engineering, economics…) and in many sub-branches of mathematics. Drawing function graphs was included as it is an application area for derivatives. Furthermore, it is also important to examine a function's behavior. As argued by (3, p.267), such examination not only provides detailed information about the current state of the action represented by the function, but it also gives information or helps making a strong prediction about its unknown (not yet realized) portion. Certain vital issues could be controlled by examining mathematical models. Examples include virus proliferation, movements of money, and price fluctuations etc.

Method

The study used the relational survey model. Relational survey is a research model that aims to determine the presence and/or degree of common changes between two or more variables. For this purpose, the researchers developed an identification test consisting of 7 questions. The test aims to identify the learning difficulties about drawing a rational function graph and derivatives. A pilot study was carried out with 98 students who took the course Analysis I during the first semester of the academic year 2008-2009.

Table 1: Item analysis for the questions

Question numbers	Item Difficulty	Item Discrimination Power
1	0.55	0.58
2	0.37	0.45
3	0.73	0.42
4	0.70	0.48
5	0.37	0.55

Item difficulty index and item discriminatory power were calculated for each question in the study. Item analysis for the questions is shown in Table 1. Two questions with item discriminatory power below .20 and item difficulty index below .29 were excluded from the study. The questions' Cronbach's alpha reliability coefficient was found to be .79. For their validity, opinions of the three field experts teaching the courses Analysis I and Analysis II were deemed as sufficient. The real study was conducted during the first semester of the academic year 2009-2010 with 84 students taking the course Analysis I. The sample contains 45 male and 39 female students who took the course Analysis I. During the main application, the students were given 50 minutes to answer 5 questions.

Findings and Discussion

Table 2: Results about question 1

	Male		Female	
Incomplete Responses and Errors	f	%	F	%
No response to the question	1	2.2	1	2.6
Incorrect derivative for inverse trigonometric function	6	13.3	12	30.8
Incorrect derivative for logarithmic function	5	11.1	13	33.3
Wrong answer although derivative rules were correctly applied	20	44.4	10	25.6

The first question given to the students was "*Calculate (y') derivative for function* $y=arctg(e^x) + ln\sqrt{\frac{e^x+1}{e^x-1}}$ ". As seen in Table 2, 13 of the male students correctly answered this question. The most common errors made by male students in this question involve simple algebra mistakes believed to be caused by lack of attention. Some of these simple algebra mistakes include writing "$1-e^{2x}$" as denominator instead of "$1+e^{2x}$" in the derivative of $arctg(e^x)$.

This was considered as a simple algebra mistake because the students made an error only during implementation after correctly writing the derivative rule for function *y=arctgx*. Another striking mistake was that the students wrote *1* as the denominator instead of e^x in the derivative of *y= arctg(*e^x*)*, which was because they simply relied on memorizing the rule. They think that the denominator should always take *1* since they did not make sense of the rule.

As clear from Table 2, the operation level at which most of the female students made errors concerned finding the derivative of a logarithmic function. 13 female students made mistakes at this level. The number of female students making simple mistakes was less than the male students (10 female and 20 male students). Such mistakes particularly stemmed from the fact that the given statement was a rational function in a root. The students made a mistake after writing the derivative of *lnu* as $\frac{u'}{u}$, which indicates that they knew about the derivative rule for logarithmic functions, but had difficulty in using it. For the statement $ln\sqrt{\frac{e^x+1}{e^x-1}}$, most of the male and female students tried to write $ln\sqrt{e^x+1}$- $ln\sqrt{e^x-1}$ before finding its derivative. However, some of the female students failed to use this rule in logarithm. This might suggest that the students who made this mistake could not learn the basics of logarithmic operations very well. The mistakes made in finding the derivative of an inverse trigonometric function were similar among female and male students.

Table 3: Results about question 2

Incomplete Responses and Errors	Male f	Male %	Female f	Female %
No response to the question	23	51.1	10	25.6
Giving a correct response without any operations while finding the derivative rule	9	20.0	10	25.6
Incorrect derivative rule found for the inverse trigonometric function	5	11.1	5	12.8
Incorrect definition or image sets for the function	5	11.1	5	12.8

The second question given to the students was *"Define the function f:x*$\rightarrow$*y=arcsinx; draw its graph and find out the derivative rule"*. This question showed that the students had shortcomings particularly in their conceptual knowledge because, as is clear from the table, 93.3% of the males and 76.8% of the females failed to provide a correct response to this question. This may be particularly attributed to focusing on operational knowledge.

Table 4: Results about question 3

Incomplete Responses and Errors	Male f	Male %	Female f	Female %
Incorrect asymptotes	1	2.2	0	0.0
Extremum points incorrectly found after finding the first derivative	11	24.4	5	12.8
Mistakes in the change table	8	17.7	11	28.2
Mistakes in transferring the data to the graph	6	13.3	5	12.8

The third question administered to the students was *"Examine the change in function y=f(x)=* $\frac{2x^2-8x}{x^2-4x-5}$ *and draw its graph"*. This question is a good example of the intertwining of mathematical subjects like the rings of a chain because students have to have a sound knowledge of functions, limit and derivatives to correctly answer this question. If a student has incomplete knowledge of any of these subjects, then it will be hard to give a correct response. More than 50% of the students in the sample failed to arrive at a correct solution. A striking point for the male students was that they failed to write the root value of the first derivative in its place in the function, a mistake which could be attributed to a lack of full understanding of the information regarding extremum points. The female students usually committed errors in forming the change table. Their most obvious mistakes were in limit operations to the right and left of critical points. Thus, other mistakes followed in subsequent operations. A conclusion we can draw from Table 4 is that female students were more successful than male students, even at a slight degree.

Table 5: Results about question 4

Incomplete Responses and Errors	Male		Female	
	f	%	F	%
No response to the question	4	8.8	0	0.0
Incorrect graph drawing	4	8.8	1	2.6
Incorrect equation written for solution	1	2.2	0	0.0
Incorrect derivative taking	12	26.6	9	23.1
Root of the first derivative incorrectly found	1	2.2	2	5.1
Mistake made after finding the side lengths of the rectangle	1	2.2	2	5.1
Solving without using derivatives to find the maximum area	2	4.4	0	0.0

The fourth question that the students were asked was *"For a group of rectangles, one corner of which is on axis X, another is on axis Y, another is on the origin and another is on line 4x+3y-12=0, how many unit squares does the one with the maximum area have?"*. From Table 5, it is clear that 4 male students and 1 female student incorrectly drew the graph for line *4x+3y-12=0*, which could be attributed to their lack of attention or nervousness. Table 5 shows that the most common mistakes of the students were made because they took the derivative incorrectly. Here, two mistakes are striking. Firstly, the students incorrectly formed the function that gives the maximum area. Secondly, after correctly finding the derivative of the function that gives the maximum area, they found the rectangle lengths incorrectly. Mistakes made after correctly finding the derivative involve simple algebra errors. 3 students made multiplication errors after correctly finding the rectangle lengths. Attention as well as content knowledge is important to arrive at a correct solution in such questions that involve multiple steps for solution.

Table 6: Results about question 5

Incomplete Responses and Errors	Male		Female	
	f	%	F	%
No response to the question	7	15.5	1	2.6
Taking the derivative incorrectly	13	28.9	6	15.3
Incorrect tangent equation	16	35.5	10	25.6
Incorrect tangent length	14	31.1	14	35.9

The fifth question was *"Find the tangent equation and tangent length of slope* $e^{xy} - x^2 + y^3 = 0$ *at point A(0,-1)"*. As revealed by Table 6, this is the question in which most male students made mistakes. Here, those who incorrectly took the derivative for the closed function naturally found the tangent equation and tangent length incorrectly. The origin of the mistakes in this question involves the derivative of closed function. The students either found the derivative of the closed function incorrectly or arrived at incorrect solutions due to simple sign errors though they found the derivative correctly. After finding the derivative correctly, a total of 16 students (4 males, 12 females) found tangent equation and tangent length incorrectly, which may have been due to lack of attention because the mistakes here originated from signs, addition or subtraction operations. Other students arriving at wrong solutions in tangent equation and tangent length did not develop a full understanding for these concepts. This is believed to be due to a lack of association formed between conceptual knowledge and operational knowledge.

Table 7: A comparison of the students' genders and their success in the derivative test

Gender	N	Mean	Std. deviation	Sd	t	P
Male	45	5.22	2.54	82	-1.33	.18
Female	39	5.90	2.02			

Table 7 shows that there is no significant difference between their gender and their success in the derivative test ($t_{(82)}$=-1.33 and p>.05). Female students were more successful than males, though the difference was insignificant.

Table 8: A comparison of the students' genders and their weighted grade point averages

Gender	N	Mean	Std. deviation	sd	t	p
Male	45	63.64	11.55	82	0.20	.83
Female	39	63.05	15.07			

As seen in Table 8, no significant difference was found between the students' weighted grade point averages and genders ($t_{(82)}$=0.20 and p>.05). Although male students had higher weighted grade point averages than females, the difference is insignificant. From the data in Table 7 and Table 8, we conclude that there is no significant relationship between the students' weighted grade point averages and their success in the derivative test.

Results and Suggestions

The students' low success rates in the derivative test and an examination of their responses to the questions revealed that a majority of them had difficulties in derivative and its applications. This result is consistent with the results of the studies conducted by (9) and (16). (9) noted that 52.6% of the students in their sample experienced difficulty in derivative and its applications. In (16) study, 54.2% of the students stated that they had difficulty in derivative and its applications. The results of the t-test revealed no significant difference between the academic success and success in the derivative test of female and male students. This result is in parallel with the result found by (2), who noted that "there was no significant difference between the academic success and success in the derivative test of female and male students".

The students' mistakes in the third question originated from a lack of understanding about fundamental subjects for derivatives and geometric interpretation of derivatives. Similarly, the students' mistakes in the fifth question were due to their inability to establish connections between conceptual knowledge and operational knowledge. These results are consistent with the result found by (2): "They failed to make a connection between geometric interpretation of derivatives and conceptual knowledge and operational knowledge".

The students found the derivatives of the functions incorrectly in questions one, two and five, which could be attributed to the fact that they attempted to memorize derivative rules, rather than learning what they mean. This result is confirmed by the results in the studies by (11) and (13). Found that the students acquired the ability to obtain the derivative of a mathematical statement given as a rule, rather than learning what derivative means (11). Demonstrated that although the students were skilled in derivatives, they relied on memorizing the formulae and use them by rote (13).

In the light of the results obtained, various suggestions are made so as to minimize learning difficulties in students and pre-service teachers and to achieve meaningful learning.

1. Rich conceptual images will contribute to students' understanding of mathematical objects (6)
2. Conceptual understanding occurs if one can establish proper connections between new and old information (12).
3. Such difficulties could be reduced or eliminated if abstract mathematical concepts are taught by using concrete tools and through concretization (5).
4. Similar studies could be conducted that examine other concepts regarding derivatives.

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Appendix 1. The Derivatives Knowledge Test

1. Calculate (y') derivative for function y=arctg(e^x) + ln $\sqrt{\frac{e^x+1}{e^x-1}}$

2. Define the function f:x $\rightarrow$ y=arcsinx; draw its graph and find out the derivative rule

3. Examine the change in function y=f(x)= $\frac{2x^2-8x}{x^2-4x-5}$ and draw its graph

4. For a group of rectangles, one corner of which is on axis X, another is on axis Y, another is on the origin and another is on line 4x+3y-12=0, how many unit squares does the one with the maximum area have?

5. Find the tangent equation and tangent length of slope e ^{xy} - x^2+ y^3 =0 at point A(0,-1)