PROBLEM BASED LEARNING IN LINEAR ALGEBRA

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Abstract

In this study, the applicability and usability of problem based learning method algebra were investigated in teaching linear. According to this purpose, it was aimed to investigate whether there is a difference between the achievement levels of students who received problem based and traditional instruction methods in learning linear combination, sub-vector spaces linear dependence-independence, the space spanned by a set and basis-dimension concepts in linear algebra course. Secondly, opinions of students in experiment group about the usability of PBL were gathered after the application. The sample of the study was composed of 34 students in experiment group and 38 students in control group, therefore, total of 72 students who are studying two different classrooms with same grade level from the department of Primary School Mathematics Education. As a data collection tool, Linear Algebra Achievement Test was used. The results of the study revealed that the instruction of related concepts with problem based learning method was more effective in increasing the achievement when compared to that with traditional instruction. In addition, students had positive opinions towards the PBL method.

Key words: *Problem based learning, Linear algebra, Teaching linear algebra.*

1. Introduction

The first studies related to mathematics education began in primary level and the necessity of doing research in university level arose due to the requirements of and in order to meet the needs of the new century (Artigue, 1999; Konyalıoğlu, 2003). The first studies done in university level began with the analysis course and the importance was given to the studies of linear algebra teaching in last two decades (Artigue, 1999; Dorier, 1995, 2002). Linear algebra does not only contribute to mathematics inside but also contribute to other science branches in both application and theoretical development of them. Therefore, linear algebra became one of the most beneficial theories both inside and outside of mathematics (Carlson, 1993; Harel, 1987, 1989a, 1989b). So, it is necessary to give attention to teaching linear algebra due to the fact that it exists in both mathematics inside and in other areas of the life.

When considering the concepts in linear algebra and the operations performed with such concepts, it is possible to state that linear algebra can be represented as the unification of the different disciplines in mathematics. The representation of the unification of the different disciplines in mathematics and its characteristics of involving different mathematical languages together (Dorier & Sierpinska, 2001; Hillel, 2000) bring both the possible opportunities and difficulties of learning and teaching linear algebra. The previous studies revealed that students generally correctly performed the operations which require calculations in linear algebra but they experienced difficulty in understanding the concepts and in constructing relations between concepts (Dorier, 1998, Harel, 1989b). In linear algebra instruction, students firstly learn the matrix operations and solution of linear algebra systems and they did not experience difficulties in doing so. However, students experienced difficulty and became mixed up in learning the concepts such as sub-vector space, space spanned by a set and linear dependence-independence (Carlson, 1993; Dorier, Robert, Robinet & Rogalski, 1994; Hamdan, 2005; Harel, 1989b). Dorier (2002) stated that students felt like they are in different planet when they faced with linear algebra.

In general, the studies that investigated the reasons of difficulties experienced in learning and teaching linear algebra revealed that the difficulties were; (*i*) abstraction (Bogomolny, 2006; Hamdan, 2005; Harel, 1989a;

Hillel & Dreyfus, 2001; Konyalıoğlu, 2009) (*ii*) insufficiencies in applying it to real life (Carlson, 1993; Dorier, 1995; Dubinsky, 1997; Fischer, 2005; Hamdan, 2005), (*iii*) inadequacy in constructing relation between previous knowledge and concepts and new (Carlson,1993; Dorier, 1998; Harel, 1989b), (*iv*) insufficiencies rooted from pedagogical approaches (Dorier, Robert, Robinet, & Rogalski 2000; Dorier & Sierpinska, 2001; Dubinsky, 1997; Uhlig, 2003) and (*v*) prejudice and negative attitude which were developed towards linear algebra (Konyalıoğlu, 2009). The proposed reasons related to learning difficulties in linear algebra were mainly based on psychological, pedagogical and epistemological factors. In his study, Konyalıoğlu (2009) determined that the problems rooted from epistemology were abstraction, meaninglessness, complexity; the problems rooted from pedagogy as memorization, lecturing style, course lecturer, inability to relate with daily life, and the classroom that the lectures were introduced. In addition, the psychological reflections of these negative epistemological and pedagogical effects were stated as fear, unlikeness, boredom, lack of self-confidence.

The studies (Dorier, Robert, Robinet, & Rogalski, 2000; Fischer, 2005; Hamdan, 2005; Harel, 2000) that emphasis the importance of constructing relation within the concepts and between concepts and real life situation in order to resolve the difficulties in learning and teaching linear algebra were in the characteristics of evoking the usage of problem based learning (PBL) method in teaching linear algebra. Harel (2000) emphasized the importance of "necessity" principle in teaching linear algebra. According to Harel (2000), the necessity principle is based on the principle of "knowledge which develops as the solution of a question". In other words, the necessity of teaching concepts is emphasized over the factors that bring students face to face with the problem situation. At this point, the situations, which PBL method brings the activities including the real life problems into forward and takes the students' previous knowledge into account, results in students' construction of their own solution methods. In addition, from these solutions, its role in helping students to construct related concepts suggests that it may be an important learning method in resolving difficulties experienced in teaching linear algebra. The second step of two level strategies, "meta lever", developed in the study conducted by Dorier, Robert, Robinet, and Rogalski (2000) shows appropriateness with the PBL's pedagogical principles. Similarly, the result of the study conducted by Fischer (2005), "students experienced difficulty in applying the mathematical concepts to real life situations", emphasized the importance of the usage of PBL method in linear algebra. In addition, the situation that the concepts in linear algebra were fundamentally based on linear equations systems (Konyalıoğlu, İpek, & Işık, 2003) and the obvious relation between linear equation system and real life support the usage of PBL method in teaching linear algebra. Moreover, the nature of concepts in linear algebra concepts and geometry which exist in its base (Konyalıoğlu, 2009) and, accordingly, the close relationship between linear algebra and geometry (Nef, 1967; Roman, 1984) and the reflections of geometry concepts in physical world in general (Konyalıoğlu, 2003) made us think that teaching linear algebra is appropriate for the usage of PBL.

PBL method is a learning mentality of how we are questioning the life and how we imitate what we learnt (Duch, 2001). PBL is beneficial not only in overcoming the difficulties but also in discovering the method followed during the process of reaching the solutions. Learning in PBL is realized over the problems. Instead of investigating only one simple solution, students interpret the problem, brings new knowledge together, define possible solutions, evaluate the options and interpret the results. In PBL method, by taking the existing knowledge related to problem into consideration, students determine what kind of knowledge they need, do research and discuss the new knowledge gained in cooperative group environment. This situation continues until finding solution for the problem (Peterson & Eaguest, 1998). This method provides students with having knowledge about what is learnt and why (Chin & Chia, 2004).

There were various studies conducted on PBL method in different disciplines. It was seen that the studies in medical and science areas towards PBL method generally took the front places. Studies in mathematics area were limited when compared to other areas. Most of the studies conducted in mathematics areas were focused generally on primary level (Akın, 2009; Akınoglu & Tandogan, 2007; Günhan-Cantürk, 2006; Günhan-Cantürk & Başer, 2009; Haris, Marcus, & Mclaren, 2001; Katwibun, 2004; Uslu, 2006). In these studies, advantages of PBL method's roles in developing students' attitudes towards mathematics, achievements and creativities rather than traditional instruction were emphasized. On the other hand, there was no study found which applies PBL method in undergraduate level, especially for the concepts in linear algebra. In this study, the foresight of applying PBL in teaching linear algebra was tried to be based on the experimental data by considering the theoretical fundaments formed under the light of literature and linear algebra's own inner structure. In this study, the purpose was to show the applicability and benefits of PBL in teaching the linear combination, sub-vector space, linear dependence-independence, space spanned by a set and basis-dimension concepts which were considered to be difficult for students to understand in linear algebra.

It was thought that the difficulties experienced in learning and teaching linear algebra can be overcome by using PBL method that necessitates the relation between the concepts in linear algebra and real life situations and that necessitates learning them through the problem proposed. It was supposed that this kind of instruction is important for university students, academicians and mathematics educators. With this aim, it was investigated that whether there is difference between achievement levels of two groups of students who received problem based and traditional instruction in learning these concepts in linear algebra.

2. Method

Both quantitative and qualitative data were used in this study. For the purpose of collecting quantitative data, quasi-experimental research design was used. According to McMillan and Schumacher (2010), this kind of research method is used in order to determine the reasons and results, and it directly affects the conditions. From the quasi-experimental methods, matched control group pre-test post-test experimental design was used in order to determine the effectiveness of two different instruction method (problem based instruction and traditional instruction methods). For the purpose of collecting qualitative data, students who received instruction according to PBL method were expected to express the positive and negative aspects of linear algebra instruction in written from at the end of the study.

2.1. Research Group

The study was conducted to two classrooms of second grade 72 students who are studying in the department of Primary School Mathematics Education in a public university in Turkey during spring semester of 2009-2010 academic years. To become primary school mathematics teachers in Turkey, students have to graduate from universities' faculty of education after receiving four-year teacher training program. Selection and placement of the students who will attend this program is based on a national university entrance exam. In this exam, the general evaluation of 12-year education from the beginning of the primary school to the end of secondary school is performed. According to the results of this exam and students' choices, students with similar achievement level are placed to the same universities' primary school mathematics education department. This department's curriculum is focused on how to teach mathematics to 12-15 age group students in general. Curriculum of Primary School Mathematics Education department includes general culture courses, courses for pedagogical and mathematical content knowledge. According to the grades gained for each course, students' general point averages (GPAs) are calculated at the end of each semester. Since students' were in the same achievement portion to enter this program and they receive GPA during the program, such information can be considered to be important factors to determine students' mathematical backgrounds.

According to the information given above, the equality of two classrooms in the study was compared with respect to the students' GPAs gained in their first and second semesters (at first grade level) before the beginning of the second grade level. There was no statistical significance found between students' GPAs in these two classrooms [t(70)=0,688; p=0,494, p>0,05]. In addition, opinions of lecturers who taught in both classrooms were also taken. Then, it was decided that these classrooms are equal. Then, these classrooms were separated as experimental and control groups randomly. There were 34 students in experiment group and 38 students in control group.

2.2. Data Collection Tools and Data Analysis

As data collection tool in this study, "Linear Algebra Achievement Test" (LAAT) developed by researchers was used. In LAAT, there were two questions for linear combination concept and one question for each of sub-vector space, linear dependence-independence, space spanned by a set and basis-dimension concepts. Since the sub-vector space, linear dependence-independence, space spanned by a set and basis-dimension concepts for which teaching activities were done in this study are included in the basis of the linear combination concept and since this concept is specially investigated in this study, this concept is presented with two questions in the post-test. According to Carlson (1993), studying these concepts with different sets necessitates different algorithms. Therefore, one of two questions prepared for linear combination concept considers the vector space, while the other considers the polynomial space.

In the first question in LAAT, the learning level of linear combination concept was tried to be measured. In this question, "By using any three vectors directed to R^3 space, is it possible to reach every point in R^3 space? Explain it with an example." was asked to students. Students' answers for whether the situation discussed is possible were evaluated out of 6 points and their explanations of their answers with example were evaluated out of 10 points. In the second question for the linear combination concept, students were asked to answer the question "whether or not the polynomial p(x)= $4x^2-2x+5$ can be written as the linear combination of $p_1(x)=2x^2+x+1$, $p_2(x)=x^2-2x+2$ and $p_3(x)=x^2+3x+6$ polynomials?".

In this question, students who could write the equality $4x^2 - 2x + 5 = a(2x^2 + x + 1) + b(x^2 - 2x + 2) + c(x^2 + 3x + 6$ for $a, b, c \in R$ by using the definition of the linear combination got 8 points. Similarly, according to this equality, the answers that indicate that p(x) polynomial can be written as the linear combination of $p_1(x)$, p_2(x) and $p_3(x)$ by calculating the constant values of a, b and c were evaluated out of 8 points.

In the second question prepared for the concept of "space spanned by a set", students were asked to write one basis for each of A= $\{(1,1,-1),(0,4,3)\}$ and B= $\{(2,1,2),(3,2,4),(1,0,1),(2,6,3)\}$ sets. In addition, they were expected to draw diagrams for these sets. Each process was evaluated out of 5 points. Therefore, the maximum score that can be gained from this question was 20 points.

The question for the linear dependence-independence concept was as follows;
"A student answered the question "Define the linear independence for vectors?" which was asked in an exam as follows; "if $a_1v_1 + a_2v_2 + ... + a_nv_n = 0$, then vector set of S= $\{v_1, v_2 ... v_n\}$ is linear independent." What do you think about this definition? Is this student's definition correct? If there are incomplete explanations, please indicate them."

The purpose of this question was to make students think the linear dependence-independence by comparing them. Therefore, it was aimed that students explain the incomplete parts of the definition. In the incomplete definition, students were evaluated out of 5 points if they correctly expressed the set from which representative elements were chosen. In the solution of the equation $a_1v_1 + a_2v_2 + ... + a_nv_n = 0$, students' answers that indicated "the linear independence can be satisfied if all coefficients of a_i should be equal to 0 (zero)" were evaluated out of 6 points.

In the question for the basis-dimension concept, students were asked to answer the question "For the sets W_1 =sp $\{(1,1,2,1),(3,1,0,0)\}$ and W_2 =sp $\{(-1,-2,0,1),(-4,-2,-2,-1)\}$, Find dim($W_1$), dim($W_2$), dim($W_1 + W_2$) and dim($W_1 \cap W_2$). Correct calculation of each of dim(W_1), dim(W_2), dim($W_1 + W_2$) and dim($W_1 \cap W_2$) were evaluated out of 4 points.

The last question asked in the question was developed to measure the sub-space concept. In this question, students were asked to answer the question "whether or not the set W= $\{(x, y, z) : x = y,\ 2y + z = 0,\ x, y, z \in R\}$ is sub-space of R^3." Students' answers to satisfy requirement of sub-space (for $u, v \in V, cu + v \in V$) for every property of the set (x=y and 2y+z=0) were evaluated out of 8 points.

During the preparation of the questions used in this study, the questions mentioned in the literature were investigated. Among those, it was decided to ask a question for each concept by considering the aim of the study. To satisfy the content validity of the questions asked in the test, the test was subjected to expert opinions of three lecturers. In addition, the pilot study of test was conducted to 72 students who are studying in the second grade level in Primary School Mathematics Education Department. These students were not included in either of experiment or control group. The result of the pilot study revealed that the reliability value of the test was 0,78. In addition, the necessary corrections on the questions were done according the students' responses to the question in the pilot study. The last version of LAAT was subject to both experiment and control groups.

At the end of the study, students in experiment groups were expected to write their positive and negative opinions towards the application of PBL methods. Therefore, students' opinions about the applicability and usability of the PBL method could be gathered. Students' answers to LAAT were analyzed by two different researchers. After the analyses done, there was a 92 % reliability found on the scores gathered from the test. When encountering different answers from the analyses of the researchers, these points were discussed by researchers together. Then, there was a consensus for the differences in the last meeting. In the study, independent t-test was used to determine whether there was a statistical significance between experiment and control group students' total scores gathered from LAAT. In addition, descriptive statistics techniques were also used to present students' scores gathered from the test. Lastly, direct quotations from students' negative and positive opinions towards PBL method were presented in the findings part of the study.

2.3. Application Steps in the Study

The concepts that were instructed in this study were performed with problem based learning method for experiment group and with traditional instruction method for control group.

The study lasted six weeks each of which includes three class hours. The average class hour in this study lasted 45 minutes. The lectures were done by the same researcher for both of control and experiment groups in the same time interval. Different than students in control group, students in experiment group were informed about the instruction processes in two class hours before the study began.

In the instruction for experiment group, PBL method's steps determined by Barret (2005) were followed in the same order. For this purpose, 34 students in experiment group were separated into seven groups, six of which included five persons while one of which included four persons. The activities prepared for each concept were distributed to the groups. Students in each group were expected to examine the problem situation mentioned in the activity and to develop alternative solutions by discussing it in the group. Then, the solutions were presented in the classroom by the representative member of each group. Therefore, it was aimed to make students discuss the important points and, if exists, the incomplete points in the solutions done by groups. Then, the concepts were tried to be constructed by emphasizing on the important points in the solutions discussed in the classroom presentations. For example, the activity shown in Figure 1 was developed for the "Space spanned by a set" concept. This activity was given to students. Groups' different solution suggestions for each question asked in this activity were discussed in the classroom. With this discussions, the fundaments of the concept "space spanned by a set" was tried to be constructed by comparing different solutions of each group in the classroom. By considering different solutions, students' answers for constructing the concept were presented in the findings part.

A pigeon is collecting food in a quadratic area with 5 meter distance through both x and y axes. The pigeon cannot go outside of this area. The pigeon can reach the points which can be constructed by different linear combinations of (1,2) and (2,4) vectors in the area and collect food from these points. So,

1. Can this pigeon reach and collect food from all points in this quadratic area? Why?
2. What is the longest distance through a line that this pigeon can collect food?
3. Choose new vector(s) for this pigeon to collect all foods in this area.
4. Discuss the relation between the vectors in the first and third questions and explain the differences that you found in these two problems.

Figure 1. The activity prepared for the "space spanned by a set" concept

In the control group, on the other hand, linear combination, sub-vector space, linear dependence-independence, space spanned by a set, basis and dimension concepts were taught with the help of traditional instruction method. In this kind of instruction, teacher is more dominant during the lesson. The instruction in control group began with algebraic definitions of the concepts. After presentation of the definitions, questions related to the concepts were solved and related theories were presented. During the solution of the problem, the visual representations were used and lots of examples were solved in the classroom.

3. Findings

3.1. The Findings for Scores Gathered from LAAT

The findings whether or not the experiment and control group students' scores were statistically significance were given in the Table 1.

Table 1: T-test scores for the differentiation of scores gained from LAAT

Group	**N**	$\overline{X}$	**T**	**P**
Experiment	34	69,67	4,874	,000
Control	38	49,94		

According to Table 1; students scores gained from the achievement test which was prepared in line with the related concepts in the linear algebra showed statistical difference according to the group (experiment-control) variable [t(70)= 4,874; p=,000; p< ,05]. According to this finding, PBL method increased students' achievements towards related concepts more than traditional instruction method did.

In the study, activities prepared for each concept were given to experiment group students. Students developed solution suggestions with these activities by discussing as groups. For example, the third activity in the Figure 1 was used for learning-teaching "the space spanned by a set" concept. A student group's solution for the first question in the activity prepared for "the space spanned by a set" concept given in Figure 2.

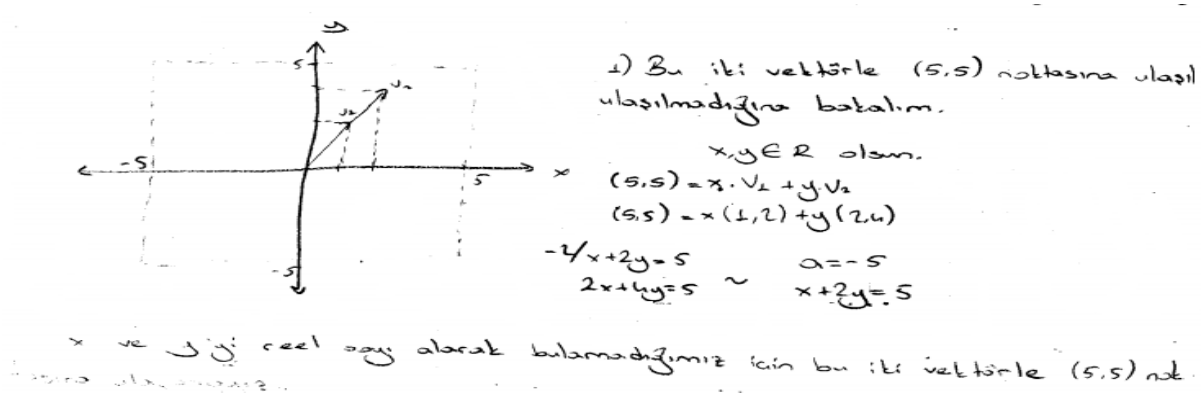

Figure 2. A solution for the first question asked in the activity prepared for the "space spanned by a set"

Note. Let's look at whether we can reach the point (5,5) with these vectors. $x, y \in R$ … Since we didn't find the real numbers for x and y, we cannot reach the point (5,5) with these vectors.

In the solution shown in the Figure 2, student group investigated whether or not they can construct any point in the defined area with the help of the different combinations of given vectors. For example, student group considered the point (5,5) and they realized that this point cannot be written with any linear combination of vectors (1,2) and (2,4). From this point, they concluded that the pigeon cannot collect all foods in defined quadratic area. Another student group, on the other hand, focused on the relations among the vectors that pigeon can move with respect to different linear combinations. Then, they realized that (1,2) and (2,4) vectors that pigeon can move with respect to different combinations were multiple of one another.

Therefore, they found that different combinations of these two vectors were lying on the same line which is $y = 2x$. So, they argued that different combinations of these two vectors can construct only the points on the line $y = 2x$ in the quadratic area. From this point, they concluded that it is impossible for the points on the line $y = 2x$ to construct a plane such as given quadratic area. The solution of the student group who gave the answer above was shown in the Figure 3 for the third question of the activity.

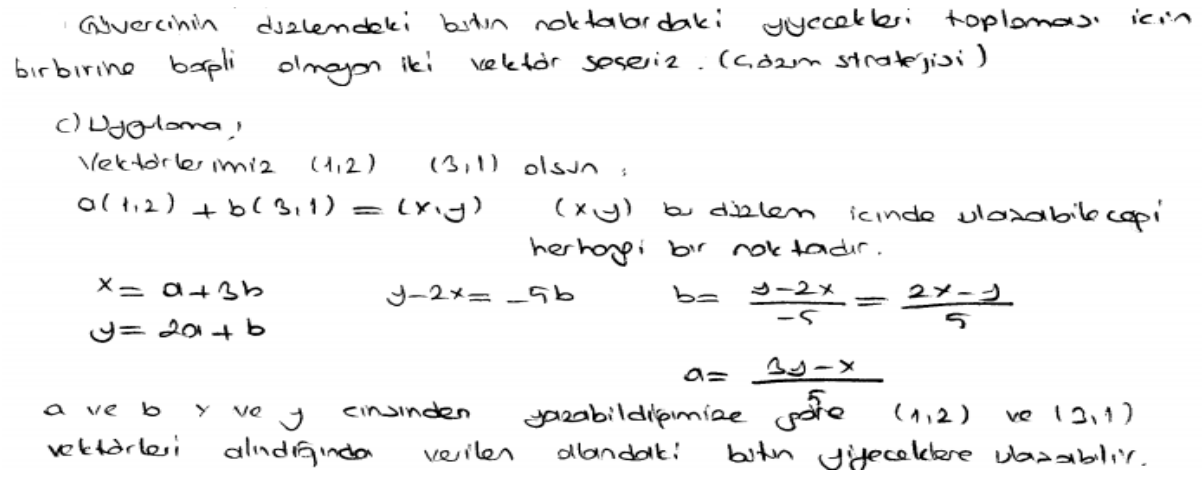

Figure 3. A solution for the third question asked in the activity prepared for the "space spanned by a set"

Note. For the pigeon to collect foods from all points in the plane, we need to choose to vectors which are independent of each other. (Solution strategy)
Application: Let's choose the vectors (1,2) and (3,1). …… (x,y) is any point that can be reached in this plane. …… Since a and b can be written with respect to x and y, all foods can be collected by taking the vectors (1,2) and (3,1) in the given plane.

Student group stated that it is enough to choose two vectors which are not the multiple of one another such as (2,1) and (3,2) to contract a plane. They tried to support this thought by writing any (x,y) vector taken from the plane with the help of different combinations of (2,1) and (3,2) vectors. Therefore, they concluded that all foods in the defined quadratic area can be collected. This student group's answer to fourth question asked in the activity developed for "space spanned by a set" was given in the Figure 4.

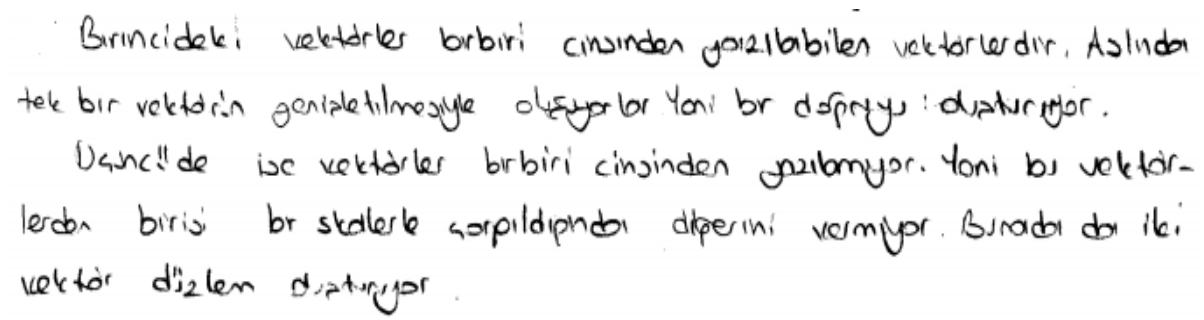

Figure 4. A solution for the fourth question asked in the activity prepared for the "space spanned by a set"

Note. Vectors in the first ones are such vectors that can be written with respect to each other. Actually, one vector is the expansion of other vector. Therefore, a line is constructed. Vectors in the third ones are, on the other hand, cannot be written with respect one another. That means, multiplying one vector with a scalar does not give the other vector. Here, these two vectors construct a plane.

In this interpretation, student group stated that (1,2) and (2,4) vectors in the first question are multiple of one another and different linear combinations of these vectors lies on the line y=2x. On the other hand, they state that (2,1) and (3,2) vectors chosen for the third question are not multiple of one another, therefore, all points in the plane can be reached with the help of different combinations of these vectors. In other words, the idea of constructing a plane was stated by this student group. Construction of this kind of results is among the objectives of problem based learning method purposes for "space spanned by a set" concept.

3.2. Experiment Group Students' Opinions towards the Instruction Performed with PBL Method

In general, students have positive opinions about applications based on problem based instruction for the linear combination, sub-vector space, linear dependence-independence, space spanned by a set, basis and dimension concepts. Relating the abstract concepts with daily life situations was seen by students as a situation that eases the learning of the concepts. One student opinion for this situation is as follows;

"With the help of the activities practiced, since the abstract subjects taught were concreted, our understandings became easy and since we found the concepts by discussing on problems, they became permanent"

Students stated opinion for the situations that previous knowledge was taken into consideration with problem based instruction; therefore, the concepts were better understood. A student's opinion toward this situation was as follows;

"From the comprehension aspect, our discovery of the concepts by relating the subjects with our previous knowledge was better"

On the other hand, students stated that they experienced difficulty because they had never been introduced with such kind of instruction method before. A student's opinion for this situation was as follows;

"PBL and group work were methods that were out of the systems we were familiar with. Therefore, it was hard for me to understand the activities at the beginning. Understanding the questions, transforming the abstract to concrete and relating it to a definition were quite hard. To do these, I had to study more. However, as the time passed, I began to be familiar with this system and to think correctly. I think this method is effective for permanent learning"

The negative opinions toward the instruction done with PBL method were generally on the group work and time problems. Three class hours for each week for teaching the concepts in the study were not seen as adequate by students in order to comprehend the concepts. In addition, the situations that such kind of learning method required group work outside the classroom by coming together and that some students did not give necessary attention to the work and that they did not follow the group rules were considered to be negative aspects of the method by students in this study. A student's opinion for this situation was as follows;

"Since problem based instruction requires group work, it is limited from some aspects. This method requires studying as a group outside the classroom; however, it is hard for everyone in the group to be in the same place at the same time. When whole group cannot be come together, the burden was put on the shoulders of a few members of the group. In addition, this method requires thinking on questions, producing different ideas and discussion. However, not every group member states opinions and goes through with these thoughts. Therefore, it requires that same individuals in the group become active all the times".

4. Discussion and Result

The applicability and benefits of PBL in linear algebra were investigated in this study. The situation that there was a statistical difference on behalf of experiment group students' scores gained from LAAT according to group variable showed that PBL method was more effective in increasing students' achievements towards learning concepts included in the linear algebra when compared to the traditional instruction method. This result supported the results of studies (Akınoglu & Tandogan, 2007; Çiftçi, Meydan, & Ektem, 2007; Deveci, 2002; Günhan-Cantürk, 2006; Gürsul & Keser, 2009; Katwibun, 2004; Uslu, 2006) done by researchers with different student levels and in different research areas.

It was observed that student groups in experiment group who received PBL method application gave different solution strategies in the problem solutions which were included in the activities prepared for each concept by them. At this point, the solutions practiced were discussed in the classroom environment; the insufficiencies realized in the solutions were stated by different groups. Sharing the solutions that student groups practiced in the classroom environment provided other student groups with opportunities of looking their own solutions from different perspective and of correcting the insufficiencies realized in their solutions. Therefore, students' construction of their own solution strategies in the activities practiced for experiment group and discussing the solutions done as a whole group could be considered to be the most important reasons for the difference occurred between experiment and control group in the Academic Achievement Test at the end of the study.

The results of study which aims to teach linear combination, sub-vector space, linear dependence-independence, space spanned by a set, basis and dimension concepts in line with the problem based instruction revealed that students generally had positive opinions for PBL. Relating the abstract concepts with daily life situations was considered as a situation that eases the understanding by students. The situations that PBL method gave opportunity for students to construct the knowledge by themselves and that it made students feel confidence to do something were among the important reasons for students to have positive opinions towards PBL method. Students' positive opinions for the utility of the PBL method supports the results of the studies conducted by other researchers (Akınoğlu & Tandoğan, 2007; Azer, 2009; Cerezo, 2004; Katwibun, 2004; Uslu, 2006).

In general, students' criticisms for PBL were on the limited time and that each student did not take enough responsibility in group works. In group works, the condition that some students did not take any responsibility outside the classroom was one of the insufficiencies observed in the study. Similar difficulties observed in the study conducted by Ertmer and Simons (2006) about the applications of the PBL method were also observed in this study.

Preparing students as producers and educating them for future successes in their lives not only depend on their knowledge of formulas and their correct calculations in the questions but also depend on the development of their mathematical understanding and mathematical thinking (Baki, 1996). To develop this kind of mentality, it is important for student to relate the concepts with real life situations and to construct relation among concepts. At this point, the situation that PBL method was more effective on students' creativity and achievement rather that the traditional method in teaching linear algebra made us think that it can be advantageous in teaching concepts in undergraduate level. In addition, this kind of teaching activity gave a general idea to pre-service teachers about the problem based learning method. In addition to the contribution to students in teaching abstract concepts in university level, it was thought that this application with PBL method will contribute students to prepare activities based on PBL method and to apply these activities prepared during their teaching lives as teachers.

Abstract characteristics of the concepts in linear algebra due to its nature hardened the direct perception of these concepts. Therefore, concepts included in linear algebra should be concretely taught. At this point, PBL method which aims to concrete the concepts and to relate them with the real life situations should not only be limited to the concepts that this study investigate, but it should be taken into consideration in teaching other concepts in linear algebra. Considering that students did not encounter with the PBL method before and that students had positive opinions about this instruction method at the end of the study, this method should be applied not only to the instruction of the concepts in linear algebra but also to that of concepts in various discipline courses.

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