

The visual representation usage levels of mathematics teachers and students in solving verbal problems

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Abstract

The present study aimed at determining the relationship between the visual representation usage levels of primary school mathematics teachers in solving verbal problems which do not include any figure, image, graphic and similar depiction and the levels of the visual representation usage in their students in similar problems. This study included 6 mathematics teachers and 121 sixth grade students who were taught by these teachers. In the study, 6 questions of the Mathematical Processing Instruments-MPI test which consist of 15 questions which was developed by Hegarty & Kozhevnikov [1] and a Likert type test which consist of 14 items were used. Consequently, a positive relationship was found between the visual representation usage levels of teachers and students in solving verbal problems. Parallel to this relationship, it was found that the students of the teachers using visual representations are frequently more successful at problem solving and that they have positive beliefs about the use of visual representations in solving verbal mathematical problems.

Keyword: Solving verbal problems, Visual representation, Mathematics teachers and students

1. Introduction

Over the past ten years, a quite different perspective on “concept” teaching has been developed and emphasized, one that differs from the traditional approaches to mathematics teaching and learning. According to the approaches based on this perspective, structuring knowledge and relating it to real life situations are of great importance. As Maccini & Gagnon [2] observed, traditional education focuses on operational calculations rather than on thinking and problem solving activities, this conflicts with newer approaches which place a greater emphasis on giving more importance to conceptual knowledge. For operational knowledge, you do not need to know the reason of a concept behind the operation you just need to know how the knowledge should be used. However, in conceptual knowledge, the focus is on comprehension [3]. The new approaches focus on conceptual learning and consider problem solving, which is a scientific method, to be the basis for mathematical activities in the developed mathematics curriculums [4]. In the process of problem solving, there is a much greater focus on conceptual knowledge. Because, individuals need to undergo a cognitive process such as understanding the problem sentence, choosing the necessary data for solution, choosing a suitable plan for solution, answering the problem and deciding if that answer is reasonable or not, expanding the problem and suggesting alternatives is essential [5].

Today, it is difficult to predict which difficulties students are going to have to face in their professional lives; therefore students should be taught how to cope with any possible difficulties they may encounter. The curriculums, which are prepared with the aim of bringing the ability to cope with difficulties through a thorough grounding in modern approaches, should aim to raise student ability in problem solving. The purpose of the problem solving approach is to enable students to think in an advanced way [6]. In mathematics, problem solving is eliminating the problem by taking the required information and subjecting it to various mental processes before proceeding with the operation [7]. The fact that the strategy of problem solving combines mathematics with daily life, introduces mathematical discipline into a group of example is key to an understanding of mathematics and increasing its value [8] and that it creates opportunities for operational and conceptual development is of great importance for students and should be taken into consideration by teachers in their classroom activities. The fact is that the studies of the process of problem solving emphasize that those who have the skills which play a role in this process are more successful, and that these skills should be the focus of mathematics education [9], shows the need for determining the choices of students related to finding solutions and in using appropriate assistive tools for problem solving.

NCTM [10] suggested that students needed to understand and improve mathematical concepts and operations. In other words, using different representations of information in classroom strengthens learning methods and improves their successes by the referral to various sources of information [4]. Representing information visually is considered an efficient representation process in mathematics education, especially in problem solving [1, 11, 12 and 13]. The importance of using visual representations in mathematics education can be explained with the contribution it makes to the development of understanding and intuitional perspectives [14, 15]. The importance given to visual representations shows itself in the visualization strategy definitions by researchers. Zimmerman & Cunningham [13] defined visualization as a skill, a product and a way of creativity and interpretation, a reflection of the diagrams, depictions, images in our minds. According to Owens & Clements [16] visualization has a significant role in understanding problems, guiding problem solving methods and it affects mental structures significantly.

The use of visual representations in the problem solving process may not always be effective and in some situations it may even lead to incorrect solutions [17, 18], however creating visual representations which emphasize spatial relationships in the process of solving mathematical problems may contribute to problem solving success. Therefore, a teaching method which is directed to create this kind of visual representations in the process of problem solving is important for students. It is known that mathematical problems have more than one solution. The solutions offered by teachers significantly affect the solutions which their students are going to use in solving similar problems. Similarly, the preferences of teachers for problem solving affects the choice of assistive instruments used in these solutions and the creation of the figures representing the situation expressed in the problem. The present study aimed at determining whether there is a relationship between the visual representation usage levels of primary school mathematics teacher in solving verbal problems (which do not include any image, figure, graphic or similar depiction) and the visual representation usage levels of students in similar problems. The sub problems of the research are:

1. What are the beliefs of students about the usefulness of visual representations for solving verbal problems?
2. What are the opinions of students about creating visual representations in solving verbal mathematical problems?
3. How are the opinions of students related to whether teachers use visual representations clearly and sufficiently in solving verbal mathematical problems?
4. Do students attempt to use the visual representations, which are used by their teachers in solving verbal problems, in solving similar problems?

2. Method

2.1. Sample: The sample of the study consists of 6 teachers working in different primary schools in the centre of Erzurum in the spring semester of 2008-2009 academic year, four of whom are females and two of whom are males and the sixth grade students who are taught by them. Sixty three of the one hundred and twenty one students who participated in the present study are females and fifty eight of them are males. While choosing the students and the teachers to include in the study, attention was paid as to whether or not they were volunteers.

2.2. Data collecting instrument: In the present study, the MPI (The Mathematical Processing Instruments) test which was developed by Hegarty & Kozhevnikov [1] the reliability and validity of which was tested and a Likert type test consisting of 14 articles was used. It was decided to perform a pilot study for the suitability to student levels, different problem solving strategies and visual representation usage. In accordance with the opinions of the experts, a decision was made to ask only 6 questions from the MPI test which consists of 15 questions. The internal reliability factor of these 6 problems chosen from the MPI test was calculated as 0.74. In preparing the Likert type test used in the research, the Likert type test which Uesaka [19] developed with the aim of determining the use of the diagram in solving verbal mathematical problems was utilized. This test which aims at determining the beliefs of students about the usefulness of visual representations in solving verbal problems and the visual representation usage levels of teachers was prepared in a way so that it could be understood by the primary school 6th grade students in accordance with the expert opinions. The internal reliability factor was calculated as 0.73.

2.3. The Analysis of the Data: In the present study, a Likert type test consisting of 14 questions and an MPI test consisting of 6 questions were applied to the students. In order to determine whether there is a relationship between the visual representation usage levels of teachers in solving verbal problems which do not include any figure, image, graphic or similar depiction and the visual representation usage levels of students in solving problems, an MPI test consisting of 6 questions was applied to the mathematics teachers of the classes it was applied to. The teachers were asked to solve the problems in the MPI test in the same way as they do in the classroom for similar problems.

From the answers given by the students in the MPI test applied, the number of the problems which were solved correctly and the number of the visual representations created by the students in solving the problems were determined. From the answers given by teachers to the MPI test, the number of the visual representations they used in solving the problems. In these measurements, if the teachers and the students create a visual representation the assessment is 1 point and if not 0 point was coded. Similarly, if the solution of the problem is correct, it was coded as 1, if not it was coded as 0. Four different measurements were recorded from the Likert type test. The first measurement was performed with the aim of determining the beliefs of students about the usefulness of the usage of visual representations in the process of solving verbal problems; the second measurement aimed at determining the current knowledge and beliefs of students about how visual representations are created in solving verbal mathematics problems; the third measurement aimed at determining whether teachers use visual representations clearly enough in solving verbal mathematical problems in their mathematics courses and the last measurement was performed with the aim of determining whether students prefer the visual representations, which are created by their teachers in solving verbal problems, while solving similar mathematical problems. The answers given to each question in the test were given points between 0 and 4.

3. Findings and Comments

In the present study, it was examined whether there is a relationship between the visual representation usage levels of teachers in solving verbal problems and the visual representation usage levels of the students in solving similar problems. When Table 1 was analyzed, a positive relationship was found between the visual representation usage levels of the teachers and the students in solving verbal problems in the MPI test. According to the result of the analysis performed, it was observed that the visual representation usage rates of the students of the teachers who use visual representations at the rates of approximately 83%, 83%, 66%, 50%, 33%, 16% (the teachers are coded as respectively (A, B, C, D, E, F) are respectively approximately 49%, 47%, 33%, 30%, 15% and 9%.

Table 1: Visual representation usage rates of the teachers and the students

	Visual Representation Usage Rates Teacher	Representation Usage Student	Problem Solving Skill Rates True	False
A	83%	49%	57%	43%
B	83%	47%	54%	46%
C	66%	33%	54%	46%
D	50%	30%	43%	57%
E	33%	15%	32%	68%
F	16%	9%	31%	69%

According to the data in Table 1, the problem solving rates of the students of the teachers who use visual representations more than the other teachers are higher as well. The line chart showing the change in the percentage of visual representation usage and problem solving success of the teachers and the students are given in Figure 1.

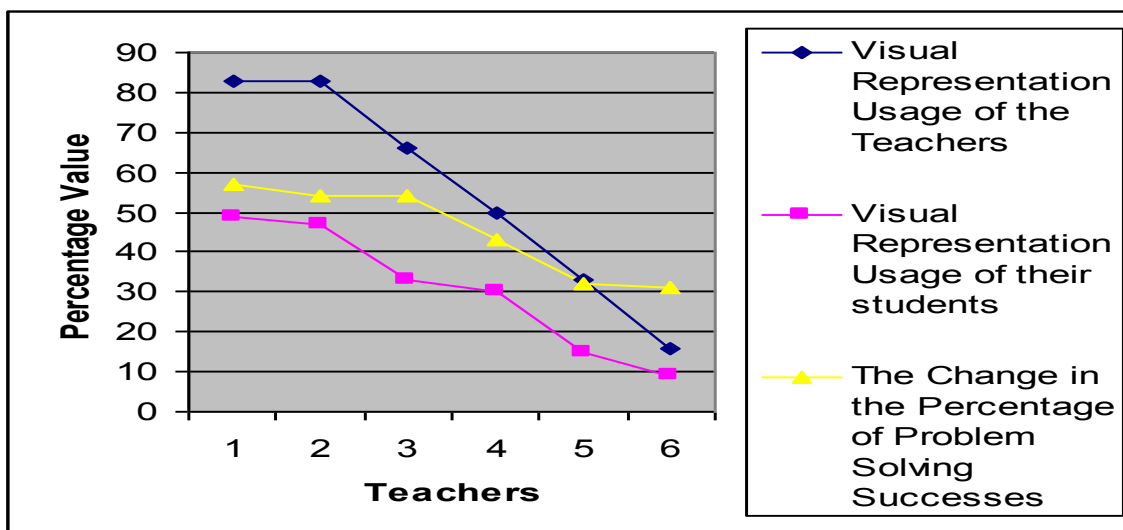


Fig. 1: Visual representation usage of the teachers, visual representation usage of their students and the change in the percentage of problem solving successes

When the data in Figure 1 are analyzed, it is seen that there is a positive relationship between the students' problem solving success and their visual representation usages. If teachers prefer visual representations in solving problems, the problem solving successes of the students increase as well. This supports the idea that visual representations can be the most important instrument for any kind of problem in all fields of mathematics including the situations which do not require geometric evidence [13]. The sample visual representations created by a teacher and his student for 2 questions chosen from the MPI test are given in Figure 2, 3 and Figure 4, 5.

Problem 2. The area of the square A is 9 square meters. One side of the square B is equal to the double of one side of the square A. Calculate the area of the square B.

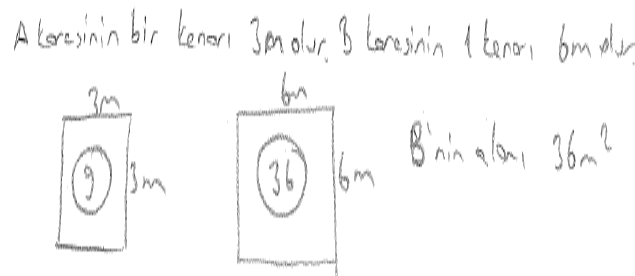


Fig. 2: Visual representation formed by teacher A for problem

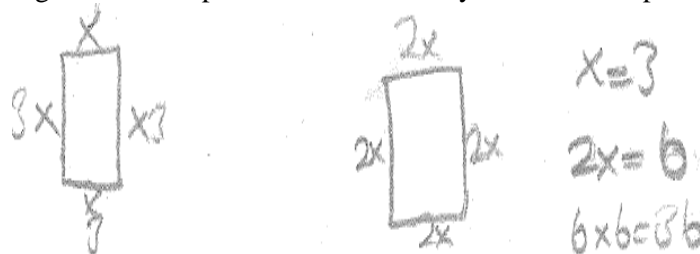


Fig. 3: Visual representation formed by the student of teacher A for problem 2

Problem 6. There are 3 jam jars and a weight of 100 gram in one of the pans of the scale. In the other pan of the scale there are two weights one of which is 200 gram and the other is 500 gram. And the scale is balanced in this way. Then, how many grams is one jam jar?

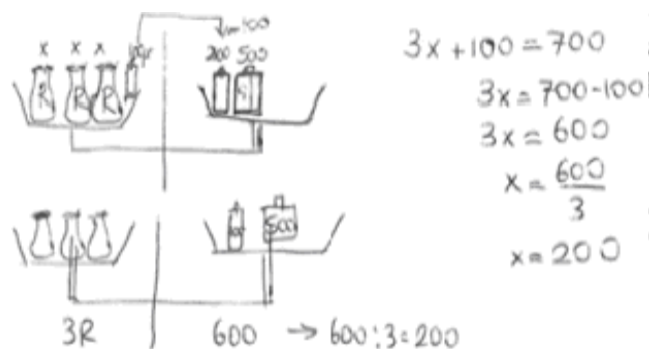


Fig.4: The visual representation formed by teacher C for problem 6

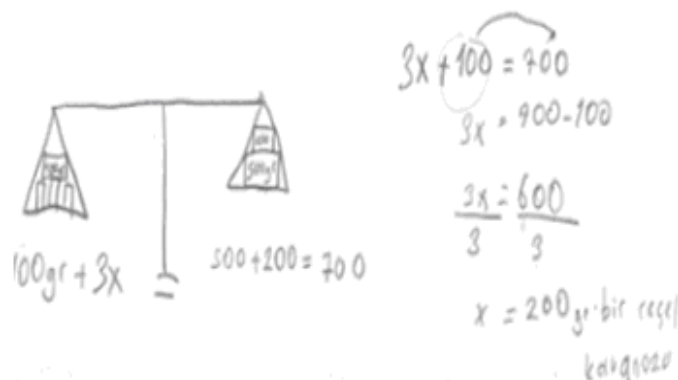


Fig.5: The visual representation formed by the student of teacher C for problem 6

Visual representation can lead to wrong solutions in the solutions of verbal mathematical problems. If the relationships between the parts in a problem cannot be successfully reflected in the visual representations; in other words, the representation does not meet the logical relationships in a problem, the representation may not lead to the correct solution of the problem. The examples which led to the wrong solution of the visual representations formed by the students for the problems 2 and 6 are given in Figure 6 and Figure 7.



Fig.6: The visual representation formed by the student of teacher A for problem 6, which leads to incorrect solution

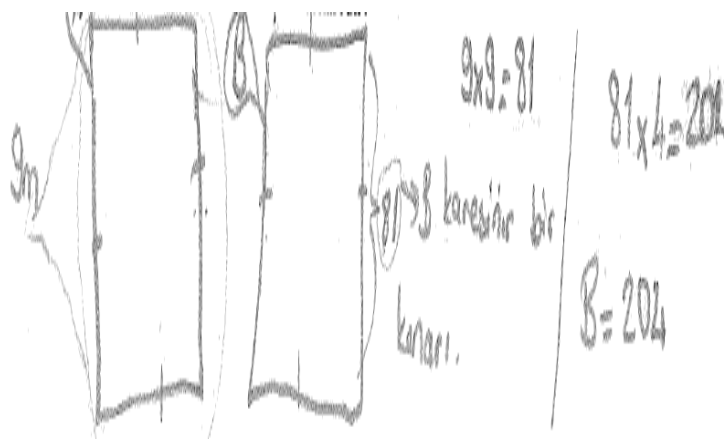


Fig.7: The visual representation formed by the student of teacher C for problem 2, which leads to incorrect solution

Table 2: The preference levels of the students for visual representations formed by the teachers for the solution of the verbal problems in the solution of similar mathematical problems

QUESTIONS	f(%)	f(%)	f(%)	f(%)	f(%)
1 In solving similar problems, I try to use the images, figures and graphics which are created by my teachers in problem solving.	37(31)	39(32)	29(24)	12(10)	4(3)
2 In order to solve mathematical problems, I use the method that my teacher uses in the same way for the other similar questions.	54(45)	38(31)	23(19)	5(4)	1(1)

In Table 2, the preference levels of the students for visual representations formed by the teachers for the solution of the verbal problems in the solution of similar mathematical problems and the answers given to the 1-2. Questions of the Likert type Test were analysed. According to the analyses, the rate of the students who answered the first question as “I always or often try to use” was 63% and the rate of the students who gave a similar answer to the second question was 76%. These rates show that the students used the visual representations, which were formed by their teachers in problem solving, in the solution of similar mathematical problems. These answers given to the first two questions of the Likert type test show that the use of visual representations by the teachers in verbal mathematical problems affected the students’ solving strategies in similar problems.

Table 3: Students’ beliefs in the usefulness of the use of visual representations in the process of solving verbal problems

QUESTIONS	Always f(%)	Frequently f(%)	Sometimes f(%)	Rarely f(%)	Never f(%)
3 I utilize images, figures and graphics in solving mathematical problems.	33(28)	35(29)	45(37)	8(6)	0(0)
4 I use the images, figures and the graphics given in books in solving similar mathematical problems.	39(32)	33(27)	35(29)	11(9)	3(3)
5 I think that using figures, images and graphics is useful for an efficient mathematical problem solving.	74(61)	24(20)	16(13)	6(5)	1(1)
6 I think that using figures, images and graphics in solving mathematical problems is a good way to learn.	68(56)	30(25)	17(14)	6(5)	0(0)
7 I think that using figures, images and graphics can be useful in understanding how mathematical problems can be solved.	66(55)	27(22)	19(16)	4(3)	5(4)

The answers given to the 3-4-5-6-7 questions of the test and the students’ beliefs in the usefulness of the use of visual representations in the process of solving verbal problems were analysed (Table 3). A great majority of the students (57%) say that they always or often use pictures, figures and graphics in solving the mathematical problems; and similarly, 81% of them think that using pictures, figures and graphics can be beneficial in solving the mathematical problems effectively. This rate shows that the students have a strong belief that using visual representation in problem solving would be beneficial. The pie chart of the students’ views related to the usefulness of visual representations in problem solving is given in Figure 8.

The rate of the students who answered the 6th question in the test as “I think it is always or often a good way” is 81.7 % and the rate of the students who similarly answered the same question is 77%. According to these rates, the students think that using visual representations in problem solving is a good way of learning and that it increases the learning success. The pie chart of these answers given to this question in the Likert type test is given in Figure 9.

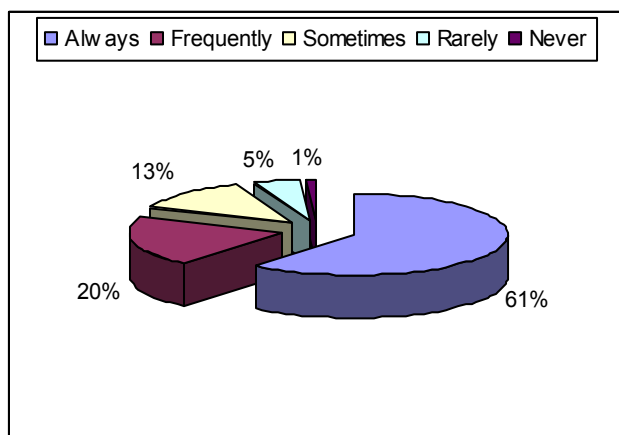


Fig. 8. The Pie Chart of the Answers given to 5th question of the Likert type test

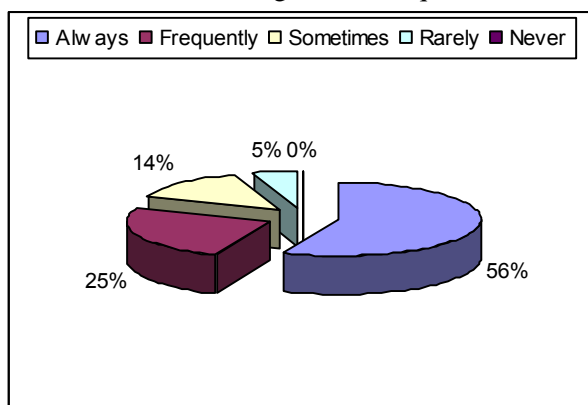


Fig. 9: The Pie Chart of the Answers given to the 5th question of the Likert type test

The rate of the answers given as “always or often” to the 4th question of the test was 59%. That the existence of a relationship between the use of visual representations and problem solving, and the high participation of the students in using the visual presentations in their book in solving similar problems, shows that visually rich source books can develop students’ problem solving skills. Visualization has an important role in the making of the problem and in leading to the problem solving method and it affects cognitive structures [Cited by 20]. In other words, forming visual representations in making sense of the problem can be a key factor. According to this premise, the use of visual representation at the comprehension stage of the problem can be effective in the solving of the problem.

Table 4: Students’ views on how visual representations will be formed in solving verbal mathematical problems

QUESTIONS	Always f(%)	Frequently f(%)	Sometimes f(%)	Rarely f(%)	Never f(%)
8 I generally know how to create images, figures and graphics in mathematical problem solving.	58(48)	38(31)	14(12)	9(7)	2(2)
9 Using images, figures and graphics in solving mathematical problems is boring for me.	16 (13)	9(7)	16(13)	26(22)	54(45)
10 It is difficult to create an image, figure or graphic in solving mathematical problems for me.	16(13)	12(10)	24(20)	26(22)	43(35)
11 I know which kind of image, figure or graphic I should utilize in solving various mathematical problems.	57(47)	32(26)	26(22)	6(5)	0(0)

In Table 4, from the answers given to the 8-9-10-11th questions of the Likert type test, the students’ views about how visual representations will be formed in solving the verbal mathematical problems were analysed. The rate of those who answered the 9th question of the test as “it is slightly or never boring for me” is 67%. This shows that portraying the data in problem solving can increase the perception of the problem; it can attract the attention of the children and thus motivate them. The rate of the students who answered the 8th question as “always or often” is 79%. This shows that the students know how pictures, figures and graphics will be formed for the solving of the verbal problems. Visual representations were classified as pictorial and formal by Hegarty and Kozhevnikov [1]. While forming pictorial representations can make the problem solvers focus on the irrelevant details, formal representations can play a more active role in problem solving [11, 12, 18]. Since the formal representations explain the spatial relationships in a better way, it increases the success at problem solving.

That the students mostly gave answers like “I know how to form visual representations” may result from the fact that the students take pictorial representations more into consideration than they do the formal representations. This is because forming an effective visual representation in problem solving means reflecting the spatial relationships in a problem with representations. This is possible with forming formal representations in problem solving. Similarly, the answers given to the question that says I know which kind of image, figure or graphic I should utilize in solving various mathematical problems” (11th question) are also high. Seventy three percent of the students say that they always or often know what kind of visual representations in problem solving is appropriate. For the 10th question, 57% of the students expressed that the use of visual representations in solving the verbal problems has never been or has been only slightly difficult for them. In the Likert type test, the pie chart of the answers given to this question is shown in Figure 10. That the students see using visual representations as an easy means in problem solving should not be ignored.

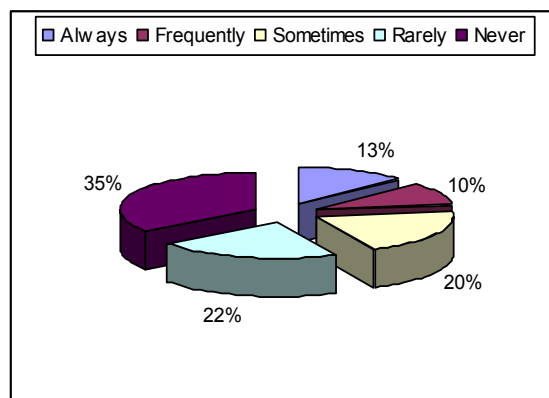


Fig. 10: The pie chart of the answers given to the 10th question of the likert type test

Table 5: Students’ views about whether their teachers use visual representations sufficiently and clearly in solving verbal mathematical problems

QUESTIONS	Always f(%)	Frequently f(%)	Sometimes f(%)	Rarely f(%)	Never f(%)
12 I can focus on the images, figures and the graphics which my teachers use in the classroom while solving mathematical problems.	70(58)	27(22)	21(17)	3(3)	0(0)
13 My teacher uses images, figures and graphics while explaining how to solve mathematical problems.	61(50)	24(20)	25(21)	9(7)	2(2)
14 The images, figures and the graphics which are used by our teachers for understanding the solution of mathematical problems help me to solve problems.	79(65)	23(19)	13(11)	2(2)	4(3)

In Table 5, from the answers given to the 12-13-14th questions of the Likert type test, the students’ views about whether their teachers sufficiently and clearly use visual representations in solving the verbal mathematical problems were analyzed. For the 13th question, 70% of the students expressed that their teachers always or often use pictures, figures and graphics in solving problems. At this point, there is a parallelism between the visual representation usage rates of each teacher and the opinions of the students about their teachers. In solving the 6 questions chosen from the MPI test, there is a general parallelism between the change in the visual representation usage rates of each teacher and the change in the rate of students’ answering the 14th question of the Likert type test as “always” (Figure 11). This verifies the hypothesis that the visual representation choices of the teachers support the students’ successes at problem solving.

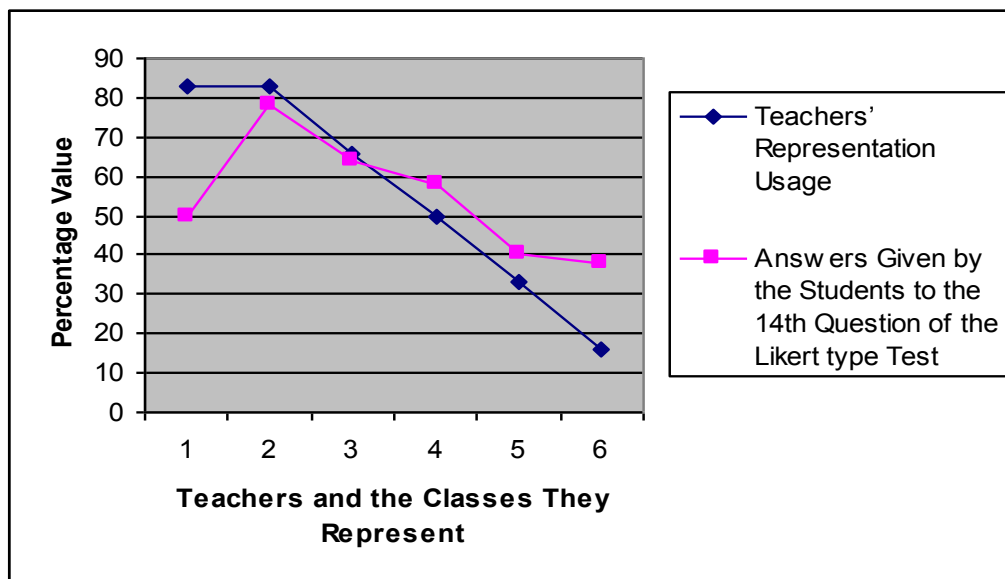


Fig. 11: The line chart of the teachers’ representation usage and the answers given by their students to the 14th question of the likert type test

Eighty four percent of the students who participated in the present study answered the 14th question of the test “it always or often helps”. That the teachers prefer visual representations in problem solving helps students to understand problems. This reflects the opinions of the students in the light of the fact that the education should be provided in line with the needs and wishes of the students which are the basic structure of the student-centred education.

The students responded to the question of “I can focus on the pictures, figures and graphics that the teachers use in solving the mathematical problems” “always or often” in the ratio of 80%. In other words, the use of visual representation in problem solving is considered as highly attention-grabbing. That the teachers prefer visual representations in solving the verbal mathematical problems could increase the motivation of the students. The pie chart of the answers given to the questions “I can focus on the pictures, figures and graphics that the teachers use in solving the mathematical problems” is shown in Figure 12.

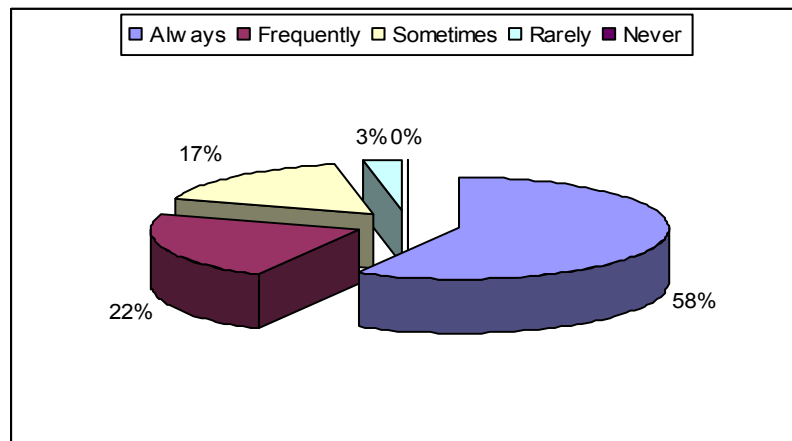


Fig. 12: The pie chart of the answers given to the 12th question of the likert type test

5. Results and Suggestions

At the end of the study, a positive relationship was found between the visual representation usage levels of the mathematics teachers teaching in 6th grades in primary schools in solving verbal problems which do not include any picture, figure, graphic or similar depiction and the visual representation usage levels of the students in solving similar problems. In addition, the present study was supported with the questionnaire developed and the opinions of the students about the use and the creation of visual representations were obtained. It was observed that the students of the teachers who rather prefer visual representations in solving verbal mathematical problems prefer visual representations in problem solving as well. A positive relationship was found between the levels of preferring visual representations and the levels of problem solving success of the students. In other words, it was seen that the students who use visual representations more in problem solving are more successful at problem solving. The findings of this study are compatible with the researches of [21, 1, 11, 12 and 13]. Hence, it can be said that when teachers prefer to use visual representations in solving verbal mathematical problems, the problem solving success of students will increase.

When the results of the study were analyzed, it was seen that most of the students do not find using visual representations boring. Therefore, teachers can make students focus on the subject by using visual representations in classroom activities. The students believe that the visual representations created are powerful assistive instruments for problem solving. To be aware of this can be advantageous for the teachers. That the students expressed that they strongly believed in creating visual representations in problem solving and they do not find using visual representations boring may make us think that using visual representations in problem solving is a success increasing factor. Therefore, the visual representations, towards which students have positive attitudes, should be much more taken into consideration.

Despite the positive attitude of the students towards using visual representations in problem solving, their problem solving successes are not very high due to the fact that visual representations are not sufficiently used in problem solving. At this point, the teachers should place more emphasis on creating schematic representations which focus on the logical relationships in a problem while solving verbal mathematical problems. Seeing that visual representations are the key component in many fields of knowledge, permanent learning can be realized thanks to visual representations. The opinions of the students about the use of visual representations and the positive relationship between the use of visual representation and problem solving shows the need for teachers to give much more time to using visual representations in problem solving. With this aim, there is a need to examine how teachers can utilize visual representations more effectively in solving verbal mathematical problems and what the qualifications for the pedagogical education to create schematic representations are, and it should be determined which kind of representations the students prefer to use.

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