# Is Partial Privatization the Optimal Choice for a Stackelberg Leader firm when there is **R&D** rivalry?

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## Abstract

This paper investigates whether a public leader firm should be partially privatized in mixed duopoly market when there is R&D rivalry. Heywood and Ye (2009) showed that the extent of partial privatization for a Cournot player firm increases in the cost of R&D parameter. That is only when R&D is very cheap should a fully public firm be retained rather than selling some of their share to the private sector. For the Stackelberg model, the relationship between the extent of privatization and the cost of the R&D is more complicated than that of the Cournot model. For higher cost of R&D the extent of privatization actually decreases.

## I. Introduction

The effect of privatization on social welfare in a wide variety of Stackelberg models in the literature showing that privatization of a public leader typically decreases welfare Benabess (2011). Matsumura (1998) considered the possibility of partial privatization in a Cournot oligopoly and showed that it is optimal for the government to sell part but not all of its shares in the public firm. Previous studies Benabess(2010a) showed that while past research would suggest that privatizing a Stackelberg leader would be unlikely to increase social welfare, that conclusion depends upon the weight on consumer surplus. When the weight on consumer surplus in the social welfare is greater than that on profit, partial privatization of the leader becomes optimal. Even more interesting, when the weight on consumer surplus is sufficiently high, the extent of privatization for the leader will actually exceed that implied for a simple Cournot firm. Thus, in some circumstances leadership may well be associated with greater privatization in contrast to the implications of previous research.

In view of the importance of technological innovation and diffusion for global economic growth and welfare, it is not surprising that social scientists increasingly devote attention to the economics of research and development (R&D). A particularly active area of research in this field in recent years has been R&D cooperation and rivalry. Pioneered by Ruff (1969), this literature had a vigorous take -off after a long dormant period, following the work of Katz (1986), d'Aspremont and Jaquemin (1988), Muller and Zang (1992).

Heywood and Ye (2009) investigated the influence of partial privatization with the presence of R&D rivalry in a Cournot model. They showed that for most reasonable R&D costs, a public Cournot firm should ignore the R&D rivalry rather than forgo privatization. They showed that only when R&D is very cheap should a fully public firm be retained rather than adopting the extent of privatization that would be optimal without the rivalry. The implication is that the privatization policy should have less importance in mixed oligopolies engaged in R&D rivalries and particularly less importance when cheap and effective R&D characterizes that rivalry.

Since our review of past models demonstrates that the most critical element in predicting the influence of privatization in mixed oligopoly is the timing of the game, which motivates me to investigate the same question for a Stackelberg model. The main goal of the present paper is to elaborate on the comparison of Heywood and ye (2009) Cournot duopoly model with Stackelberg mixed duopoly model. In other words, I am asking whether the optimal extent of privatization of a public Stackelberg leader increases in the presence of R&D rivalry. This paper is organized as follows; section 2 presents the model with the third section presenting the equilibrium. The major results on the influence of privatization are in the fourth section. The fifth section concludes the paper.

## II. Model

We consider a duopoly in which firm 1 is a public firm (Stackelberg leader) maximizing social welfare and firm 2 is a private profit -maximizing firm (follower).

The firms face a common demand function p = a - Q where a > 0 and  $Q = q_1 + q_2$ . Each firm has a production  $\cot C_i(q_i, x_i, x_j)$  that is a function of its own production, the research it undertakes  $x_i$ , and research that its rival undertakes  $x_j$ . Following the tradition from the mixed oligopoly literature, we assume quadratic production  $\cot C_i = q_i^2 - (x_i + \beta x_j)q_i/2$ . As in D'Aspremont and Jacquemin (1988), increasing R&D serves to lower production costs. Thus,  $0 \le \beta_i \le 1$  for i = 1 or 2 measures the degree of R&D spillover from the rival. Following Heywood and Ye (2009) model, we assume that  $\beta_1 = \beta_2 = 1$ ; that means that there is complete spillover and the R&D of both firms are publicly known and of equal value to both firms.

Following D'Aspremont and Jacquemin (1988), the direct cost of R&D is also quadratic, reflecting diminishing returns to R&D expenditures. Firms choose a level of R&D and a subsequent output level. The profit function of firm i is:

 $\pi_{i} = pq_{i} - C_{i}(q_{i}) - Ax_{i} \qquad i = 1, 2$ Where *A* is the cost parameter of R&D. The resulting social welfare function becomes:  $W = 0.5(q1 + q2)^{2} + \pi_{1} + \pi_{2}$ (2)

The objective function of the private firm (firm2) is to maximize(1),  $\pi_2$ . The objective function of a public firm or partially privatized firm (firm 1) is to maximize the following:  $G = (1 - \alpha)W + \alpha \pi_1$ (3)

Where  $0 \le \alpha \le 1$ .  $\alpha$  is the extent of privatization. Matsumura (1998) defines this parameter as the share of a previously public firm sold to private sector.

Thus when  $\alpha = 0$ , the fully public firm maximized social welfare from (2). When  $\alpha = 1$  the public firm maximized its profit and becomes fully private.

We consider a three-stage game, where the second and third stage consists of two steps (firm 1 acts first then it is followed by firm 2).

In the first stage, the government adopts the extent of privatization,  $\alpha$ , that maximizes social welfare (2). In second stage, the public Stackelberg leader (firm 1) chooses a level of R&D first followed by firm 2. In stage 3, firm 1 takes into consideration the chosen level of R&D and chooses an optimal level of output and firm 2 observes the level of output chosen by firm 1 and then chooses its level of output. The Nash equilibrium is derived by backward induction.

# III. The Equilibrium

In solving for stage three, the R&D levels are assumed given and mutual best response

$q_1$	_	$(11+\alpha)(2a+x_1+x_2)$		(4)	
	_	$2(43+13\alpha)$		(+)	
<i>q</i> <sub>2</sub>	=	$(8+3\alpha)(2a+x_1+x_2)$	(5)		
		$2(43+13\alpha)$	(5)		

The quantities are now functions of R&D levels and the extent of privatization. To solve for stage two, these quantities (4) and (5) are returned to the objective functions from which mutual response functions in R&D levels are derived and again firm 1, Stackelberg leader, acts first followed by firm 2.

$$x_{1} = \frac{-2a((-1669647 + 437095 \alpha^{2} + 416715 \alpha^{3} + 104780 \alpha^{4} - 878275 \alpha + 8788 \alpha^{5})A + 8192 + 4096 \alpha - 5184 \alpha^{3} - 5376 \alpha^{2} - 1566 \alpha^{4} - 162 \alpha^{5})}{\delta}$$
(6)

$$x_{2} = \frac{4a(3042.\alpha^{4}A - 81\alpha^{4} + 36348A\alpha^{3} - 864\alpha^{3} + 162242A\alpha^{2} - 3456\alpha^{2} + 320608\alpha A - 6144\alpha + 236672A - 4096)}{\delta}$$
(7)

Where  $\delta = 16384 + 16384\alpha - 162\alpha^5 + 27350408A^2 + 1536\alpha^2 - 3456\alpha^3 - 1404\alpha^4 - 2616335A + 271323\alpha^3A + 92612\alpha^4A - 2160707\alpha A + 8788\alpha^5A + 14999088A^2\alpha^2 + 3023072\alpha^3A^2 + 228488A^2\alpha^4 + 33074912\alpha A^2 - 211873A\alpha^2$ 

The R&D levels from Equations (6) and (7) can be returned to the expressions in equations (4) and (5) to yield the equilibrium levels of output.

$$q_{1} = \frac{4aA(-117\alpha^{4} + 4394\alpha^{4}A - 2298\alpha^{3} + 91936A\alpha^{3} - 14017\alpha^{2} + 623844A\alpha^{2} - 34608\alpha}{\delta}$$

$$q_{1} = \frac{4aA(-351\alpha^{4} + 13182\alpha^{4}A - 3969\alpha^{3} + 165958A\alpha^{3} - 16776\alpha^{2} + 781482A\alpha^{2} - 31424\alpha}{+1630818\alpha A + 1272112A - 22016.)}$$

$$(8)$$

δ Thus, all equilibrium choices are function of alpha, the extent of privatization. I then substitute the four equations above (6), (7), (8) and (9) into the welfare function (3), and derive the optimal extent of privatization and make critical comparisons.

Solving  $\frac{dW}{d\alpha} = 0$  yields the optimal extent of partial privatization, but the resulting expression cannot easily be analyzed. It is a function of A.

#### **IV. RESULTS**

 $q_{2} =$ 

First, I investigate the impact of the presence of R&D on the extent of privatization.

**Proposition 1:** 

For a higher R&D cost parameter, the optimal extent of partial privatization for the public leader firm actually decreases.

Proof:

Solving  $\frac{dW}{d\alpha}$ =0 yields the optimal extent of partial privatization but the resulting expression cannot be easily solved for  $\alpha$ . To give a flavor of the magnitude of this relationship, I can examine the relationship between  $\alpha$  and A. Figures 1 and 2 graph that relationship over the permissible range of  $\alpha \in [0,1]$ .

The proposition 1 shows that when the cost of R&D is high, the Stackelberg leader public firm sells less of its shares to the private sector; which is the opposite of the Cournot model that Heywood and ye (2009) derived. They showed that the extent of privatization actually increases in R&D cost.

#### Figure 1: The relationship between the optimal extent of privatization and the R&D cost parameter $A \in [0, 1, 5].$



Thus, for high values of A, it would be optimal for the public Stackelberg firm to focus on the rivalry and ignore privatization. The intuition is that privatization reduces both the cost asymmetry and the total amount of R&D. These have dominant effects on overall costs and so welfare.

Α

Figure 2. The Relationship between the optimal extent of privatization and the R&D cost parameter  $A \in [0, 12]$ .



Figure 2 shows clearly the negative relationship between A and a.

The intuition behind proposition 1 is as follows: the optimal A cost of R&D no longer leads to symmetric outputs equilibrium ( it no longer achieves the first best quantities), instead the private leader' output exceeds that of any follower, i.e. the privatized leader produces where MC>P and the followers produce where P>MC. The cost of R&D decreases with privatization. R&D will remedies the low output of imperfect competition but it will not restore the cost inefficiency as it did in public leadership, because private leader will always produce more than any follower and have a higher MC.

### V. Conclusion

This paper contributes to the growing literature on R&D and has twin goals. Its main purpose is to investigate the effect of the presence of R&D on the optimal extent of privatization. A secondary purpose is to elaborate a comparison between the models: Cournot derived by Heywood and Ye (2009) and the Stackelberg model. I have showed that the relationship between the cost of R&D and the optimal extent of privatization of a public Stackelberg firm is little more complicated than the one of Cournot model. When the cost of R&D is high enough, the optimal extent of privatization is very minimal. These results confirm that Stackelberg is associated with nationalization rather than privatization. Since both the optimal extent of privatization for Stackelberg and Cournot duopolies are complicated to analyze and long, I couldn't compare the two. For a given A, which public firm would sell more if its share to the private sector; The Cournot player or the Stackelberg leader? This question can be investigated for future research.

### References

- D'Aspremont, C. and Jacquemin, A. (1988). "Cooperative and Noncooperative R&D in Duopoly with spillovers", *American Economic Review*, Vol. 80, pp.1133-1137.
- Benabess, N. (2010). "Privatization and Subsidization in Mixed Oligopoly", *Global Journal of Management and Business Research*, Vol. 10, issue 6, pp.14-18.
- Benabess, N. (2011). "Privatization in Closed and Open Mixed Oligopolies", Journal of American Academy of Business, Cambridge, Vol. 16, pp.1-8.
- Heywood, S. and Ye, G. (2009). "Partial Privatization in a Mixed Duopoly with an R&D Rivalry", *Bulletin of Economic Research*, Vol. 61, pp. 165-178.
- Katz, M. (1986). "An Analysis of Cooperative Research and Development", Rand Journal of Economics, Vol. 17, pp. 527-543.
- Matsumura, T. (1998) "Partial Privatization in Mixed Duopoly", Journal of Public Economics, Vol. 70, pp.473-483.
- Muller, E. and Zang, I. (1992). "Research Joint Ventures and R&D Cartels", American Economic Review, Vol. 82, pp.1293-1306.
- Ruff, L. (1969). "Research and Technological Progress in a Cournot Economy", Journal of Economic Theory, Vol. 1, pp. 397-415.