Product Warranties under Ambiguity

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Abstract

This study analyzes warranties as a marketing mechanism in a framework in which consumers face ambiguity about product quality. Producers use warranties to induce ambiguity-averse consumers to participate in their product markets. Product prices and product warranties are determined in equilibrium, incorporating market participants' beliefs about product quality. Competition among customers induces producers to improve product quality, leading to long duration for product warranties.

JEL classification

Keywords: Heterogeneous beliefs, sophisticated consumers, unsophisticated consumers, Ambiguity aversion; Ellsberg paradox; Warranties; Product quality

1. Introduction

This paper proposes a theory of product market participation based on a market mechanism that induces participation by consumers who are averse to ambiguity.[1] In the economics literature; it is known that consumers are willing to pay a premium for brand products. Several explanations have been offered for this consumer behavior, different search costs incurred by informed and uninformed consumers to determine the quality of product (Bronnenberg, Dube, Gentzkow, and Shaprio, 2015) and aversion to ambiguity about product quality (Khaled and Muzere, 2017). This raises the question of how producers can increase participation in their product markets. Khaled and Muzere argue that branding of products reduces ambiguity about product quality leading to increased participation by ambiguity-averse consumers. In this paper, I argue that, in the markets for durable goods, product warranties are a market mechanism that induces ambiguity-averse consumers to participate. Product warranties are priced in equilibrium, offering useful insights about product warranties offered by producers of consumer products.

The market is characterized by monopolistic competition in durable products like automobiles, computers, and household appliances such as washing machines. Producers and consumers have different beliefs about the quality products they produce. There are two types of consumers: sophisticated consumers and unsophisticated consumers.[2]

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[1] Ambiguity refers to uncertainty about the "true" probability distribution governing future outcomes of a random event. The decision makers attitude toward ambiguity determines how and what extent such uncertainty affects the decision the agent's choices, whether the agent is averse to such uncertainty, and if so, the level of aversion (Collard, Mukerji, Sheppard and Tallon, 2011)

[2] Like Kalay (2012), I use the term "sophisticated investors" in a broad sense. This broad definition of sophistication is meant to capture an investor's ability to utilize disclosed information to gain an informational advantage. It describes active information users who devote more time and attention to their investments and are more proficient in analyzing investment-related information. Differences in sophistication can arise from heterogeneity in investors' opportunity cost of time and in their ability to acquire and analyze information. Consequently, sophisticated information processors are apt to learn more from available information.

Consumers purchase products to maximize their expected utility derived from the quality of the products. Producers choose quantities of the products to produce and warranties to support their products to maximize their expected profits.

First, I obtain analytical representations for product warranties, prices, products purchased by consumers. The economic benefits of product warranties are incorporated into product prices, which are influenced by market participants' beliefs about the quality of products.

Second, the design of product warranties is influenced by beliefs of about product quality by the market participants. When consumers' aggregate beliefs about the quality are low, they purchase longer duration product warranties because they expect to face many product repairs.[3]

Third, the costs of product warranties are included in the prices of the products. Consumers are willing to purchase the warranties because they provide consumers with protection against expenses incurred in servicing product breakdowns. Additionally, producer provided warranties are relatively cheap compared to obtaining insurance from third party insurance companies. The logic is that insurance companies have their own beliefs in product quality, and would increase their profit margins in the purchase of parts and labor relative to those of producers. If the cost of producing replacement items is low, producers offer consumers long duration warranties.

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See, for example, Fischer and Verrecchia (1999).

[3] During the Apple iPod praise many competing firms launched new products to share Apple's pie. They even lowered their price substantially compared to the price of iPod but still the iPod continued to outsell the competition despite the better quality alternative products. The explanation seems to be that beliefs about product quality and product warranties influence consumers' decisions to participate in a product market especially markets for new products from unknown firms. For Apple iPod warranty reference see http://.www.apple.com/legal/warranty/.

A producer's output of a product is positively related to consumers' beliefs in the quality of the product. If consumers on average have low beliefs in the quality of a product, the demand for that product is low. As a result, the producer loses some share of the market. This has important implications for producers who want to remain competitive in the industry. This suggests that increasing product quality is key to improving the perceptions of consumers about the quality of a product, consequently the demand for the product. Thus, competition has indirect, positive impact on warranties through improving product quality. The rest of the paper is organized as follows. Section 2, provides an overview of product warranties. These include marketing, economic, and legal functions of product warranties. Section 3, describe the model and provide the results. Section 4, provides the effect of consumers' beliefs in product price, and the effect of consumers' beliefs in product warranty. Section 5provides a summary of the results and proofs of all technical results are provided in the Appendix.

2. Warranties: An overview

2.1 *Marketing Functions of Warranties*

Consumers are often uncertain about the quality of a product and the extent to which the product will render them the services expected of the product. Thus, in a world with imperfect information, producers search for mechanisms to market their products to consumers. These mechanisms include reputation (Shapiro, 1982), advertisements (Nelson, 1974; Kihlstrom and Riordarn, 1984; Milgrom and Roberts, 1986), warranties (Spence, 1977; Gal-Or, 1989), risk sharing (Heal, 1977), and product-specific investments (Klein and Leffler, 1981). Product warranty plays an important role in the creation of market for the product and for managers to manage the market. This is due to consumer uncertainty regarding product performance over its life cycle. Warranties serve as persuasive marketing tools: promotional and protectoral. As a promotional tool, warranties serve to promote the reliability and quality of a product with longer and better warranty terms which imply that the product is more reliable. On the other hand, as a protectoral tool, warranties provide assurance to consumers against defectiveness of products that fail to perform satisfactorily over the warranty period. This assurance reduces the risks associated with purchase of the product. It is argued that if a manufacturer offers a better warranty than a competitor, then the reliability of the product should also be better to reduce costs associated with warranty claims. Therefore, warranty is an important product feature and can be used by marketing to promote sales.

2.2 Economic Functions of Warranties

Four theories have been proposed in the literature on economic rationales of warranties. The insurance, signaling, sorting, and incentive theories are based on the premise that warranties lower the risk associated with a purchase, whereas, the sorting notion is driven by firms' desire to extract consumer surplus. Insurance theory is based on the premise that warranties provide insurance to customers and work as a risk-sharing mechanism because they oblige the manufacturer or seller to compensate the buyer in the event of product failure (Heal 1977). According to Chu and Chintagunta (2011), the key assumption underlying the insurance theory is that consumers are risk averse. In the absence of risk aversion, there is no room for insurance to play a role because risk-neutral or risk-loving consumers can simply bear all the risks. Therefore, providing insurance to buyers appears to be the most fundamental function of warranties. They also argue that direct implication of the insurance rationale for warranty provision is that the degree of risk aversion and duration of warranty positively correlated. Therefore, it is expected that the same customers will buy longer warranties when product failure increases and reliability decreases, and given a particular product failure rate, more risk-averse customers will buy longer warranties than less-averse customers.

Chu and Chintagunta (2009) applied BLP to quantify the economic value of warranties in the U.S. server market. They found that manufacturers and downstream firms of the indirect channels benefit from warranty provision and from sorting across heterogeneous customers by offering a menu of warranties. Choi and Ishii (2010) sought empirical evidence of the role of warranties as signals of unobservable quality. They adapt the linear random utility model of consumer automobile demand to investigate into the extent to which warranties affect consumer choice and the extent to which this estimated warranty affect is due to risk aversion and signaling motives. The signaling theory is based on the premise that warranties are used to signal product quality to buyers. That is, producers use warranties to signal the quality of their products. Spence (1977), Gal-Or (1989) analyzed the use of warranties as a mechanism to signal product quality. Spence shows that warranties serve as a perfect signal in competitive markets because the cost of warranties increases if the product is likely to break down. Gal-Or argues that warranties serve as a perfect signal in oligopolistic markets only in the cases where product attributes are neither too clustered nor too widely spread. Otherwise multiple separating equilibriums may arise, limiting the information content of warranties.[4] According to Chu and Chintagunta (2011), the key assumption of signaling theory is information asymmetry in the sense that sellers need to have better knowledge about the product quality than buyers. Hence, it is expected that buyers may not be adequately informed about the product performance before purchase, so they try to assess this from its price and/or warranty (Spence, 1977). In addition, warranties work as incentive mechanism for producers to reveal and improve product quality. Grossman (1981) analyzes the incentive aspects of warranties. He argues that warranties provide producers with incentive to improve product quality. This theory implies quality signaling of warranties implies a positive relationship between product quality and warranty duration because only high-quality businesses can afford long warranties because of the associated costs of warranty fulfillment. Choi and Ishii (2010) argued that the assumption that a warranty could be a credible signal of unobservable product quality only if the warranty is relatively more costly than firms producing low quality products. The sorting theory of warranties is based on the premise that warranties are used as a screening mechanism in markets characterized by consumer heterogeneity in quality evaluation and risk attitude (Kubo 1986; Matthews and Moore 1987; Padmanabhan and Roa, 1993). The key assumption of this theory of warranties is the presence of consumer heterogeneity. Consumers differ in their level of evaluation of a product's quality and in their level of risk aversion.

[4] In addition to product guarantees as a mechanism to signal product quality, the information asymmetry literature uses advertisements to signal product quality (Nelson, 1974; Kihlstrom and Riordarn, 1984; Milgrom and Roberts, 1986; Noll 20110).

This is considered as private information not observable by producers. As it is often impossible for manufacturers to design a contract for each individual consumer, firms will design a contract for each type of consumers and let consumers self-select into different contracts. Therefore, equilibrium firms will offer a line of products distinguished by different quality, warranty, and price levels. Therefore, sorting theory of warranties plays a role in a given market if one observes different combinations of these three attributes in the data. Also, the sorting theory implies that in equilibrium, in response to the menu of warranties provided by the manufacturer. Customers, with the same observable attributes but different degrees of risk aversion, will exhibit different choice behaviors for warranty contracts in accordance with their risk profiles. Incentive theory states that warranties

work as an incentive mechanism for firms to reveal and improve product quality. The key assumption is risk endogen in the sense that sellers' actions can affect product performance. The probability that a product will break down is a function of its quality which depends on the producers' quality efforts and also on the consumers' maintenance efforts. Chu and Chintagunta (2011) empirically investigated the implications of this theory in the U.S. server and automobile market, namely that quality is negatively correlated with current warranty and quality is positively correlated with past warranties.

2.3 Legal Functions of Warranties

Warranties protect producers against lawsuits. It is a guarantee that the manufacturer of a product will repair damage for free for a certain period of time. The contents of written warranties on consumer products are covered by the federal Magnusson-Moss Act of 1975. The law divides warranties into express and implied warranties. Durations of warranties may vary considerably, depending on the type of transaction and warranty involved, and the applicable law. In most states, one has up to four years to enforce an implied warranty after the start of the transaction.

Where an express warranty arises because of some action on the part of the seller other than entering into a sale, an implied warranty arises from the mere entering into a sales agreement. The general approach of an implied warranty is to give the buyer some quality protection. On the other hand, contracts are ubiquitous in today's business environment, often governing almost every aspect of a commercial transaction. As a standard part of these contracts, warranty clauses play a critical role in allocating risk and determining the final sale price. Therefore, a consumer will often insist on the inclusion of certain seller warranties in the contract. Hence, these seller warranties can serve many functions, but their main purpose is usually to give assurances to the buyer about the object of the purchase. On the other hand, the seller will typically ask for certain buyer warranties, which usually appear in the form of reliance waivers. These waivers assure sellers that they are not exposing themselves to liability for false or misleading representations.

3. The Model

The market is characterized by monopolistic competition. The use of producers of products in a broad sense means manufacturers of products and distributors of the products as well. The reason is that distributors of products often offer consumers insurance for the products they sell in addition to warranties provided by manufacturers of the products. Distributors are often the ones who implement manufacturer -provided warranties for the products they sell. For example, automobile dealerships implement factory warranties, making such repairs as warranted by the producers. These warranties are bundled together into warranties that support the products distributors sell. Manufacturers tend to have several lines of products and distributors who often deal with many manufactures. I assumed the abstract from this reality by assuming that each producer has one product, simplifying producers' profit functions. Producers produce products of different qualities. Even if the producers use the same technology to produce products, their quality control mechanisms are sufficiently imperfect. Producers who are uncertain about the quality of the products they produce make consumers feel ambiguous about product quality. As a result, producers and consumers form expectations (beliefs) about the quality of products in the markets. Table 1, provides descriptions of the symbols that are used in this analysis.

Table 1: List of symbols with definitions

Symbol	Description
E	Expectation operator
var	Variance operator
$\tilde{X}_j$	Exponential random variable with parameter λ_j representing producer $j's$ quality (durability)
N	Number of producers in the industry
H	Number of consumers in the economy
γ	Coefficient of absolute risk aversion
m_j	True mean of the quality (durability) of the product of producer j
v_j	True variance of the quality (durability) of the product of producer j
q_j	Quantity of the product produced by producer j

p_j	Price of the product of producer j
T_j	Duration of the producer $j's$ product warranty
η_j	Expected durability for producer $j's$ product with a warranty
t_j	Expected number of replacements of parts during the time period of the warranty for producer j
m_j^i	Beliefs of consumer i about the mean of the quality (durability) of producer $j's$ product
v_j^i	Beliefs of consumer i about the variance of the quality (durability) of producer $j's$ product
x_j^i	Quantity of producer $j's$ purchased by consumer i
a^i	Amount of funds in $i's$ savings account
$\tilde{V}$	Monetary value of product durability
α^i	Proportion of consumers of type i = s, u
$\tilde{W}$	Consumer budget constraint

3.1. *Producers*

There are $N>1$ producers in the market. The focus is on an industry that produces durable products like automobiles, computers, household appliances, etc. Product quality depends on a number of features such as durability.[5] Following Gal-Or (1989), the researcher has used the durability of a product as an approximation for the quality of the product. Producers provide consumers with warranties to induce consumers to participate in their product markets. The strategy is to increase the sales of their products and avoid issues like law suits about defective products.[6] The warranties are commitments made by the producers to replace broken items with new ones during the time periods specified in their warranties. If items break down after the expiration dates, consumers have to purchase new items.[7]

[5]For example product quality is a function of durability and features like quietness of appliance motors, miles per gallon of gasoline and safety features like air bags of automobiles. For computers, product quality is a function of durability, ram storage, speed, versatility, and protection from viruses in the Internet.

[6] In 1998, Hyundai began to overhaul its image in an attempt to establish itself as a world-class brand. Chung Ju Yung transferred leadership of Hyundai Motor to his son, Chung Mong Koo, in 1999.

Hyundai's parent company, Hyundai Motor Group, invested heavily in the quality, design, manufacturing, and long-term research of its vehicles. It added a 10-year or 100,000-mile (160,000 km) warranty to cars sold in the United States and launched an aggressive marketing campaign.

[7] Partial guarantees could be considered, but I focus on complete guarantees for simplicity.

A warranty prolongs the life of a product, thus increasing the quality of a product with a warranty. Product warranties are attractive to consumers for two main reasons: First, warranties protect consumers against the costs of repairs to their acquired products. Second, producer provided warranties are cheaper than insurance companies provided warranties. Another reason is that insurance companies form their own beliefs in product quality. They add profit margins to parts bought and labor provided, thus increasing the cost of product insurance. The use of an exponentially distributed random variable is to represent the actual durability of a product. An exponential distribution is particularly suitable for handling repairs that occur during a product warranty because an exponential random variable does not have memory. The researcher provides some information about an exponential random variable.[1]

[1]Let $\tilde{X}$ denote an exponential random variable with parameter λ. The probability density function for the random variable $\tilde{X}_j$ is defined as

$$f(t)=\begin{cases}\lambda_j e^{-\lambda_j t} \text{ if } t>0\\ 0 \text{ otherwise}\end{cases}$$

Let $\tilde{X}_j$ denote the durability of producer $j's$ product which is assumed to be exponentially distributed with parameter $\lambda_j = \frac{1}{m_j}$ where m_j and v_j are the mean and variance of $\tilde{X}_j$, respectively. For simplicity, I assume that random variables $\tilde{X}_j$ are mutually independent, where $j = 1,...,N.$

3.2. *Consumers*

There are $H > 1$ consumers, sophisticated and unsophisticated. Let α^i denote the proportion of consumers of type i = s, u where $\alpha^s + \alpha^u = 1$ and $\alpha^i \in (0,1).$ Consumers are ambiguous about product quality. Put another way, consumers are ambiguous about the durability of products in markets. There are several ways to formulate ambiguity aversion.[8] The adoption of the Gilboa and Schmeidler (1989) formulation of ambiguity aversion in which consumers who are averse to ambiguity are greatly influenced by the worst-case scenario. These consumers have max-min utility functions.

3.3. Beliefs about Product Quality

Hence, Easley and O'Hara (2010) in constructing the beliefs of market participants about product quality. Although producers are uncertain about product quality, their beliefs are specified by the true parameters of the distributions of the durability of their products.

[8]Schmeidler (1989) relaxes Savage's 91954) independence axiom and defines expected utility as the Choquet integral of utility function with respect to investor's beliefs. Beweley (2002) considers decision makers with incomplete preferences. In this theory, one portfolio is preferred over another if and only if it yields a larger expected utility for every belief in the set of beliefs used to represent the individual's preferences. Ghirardato, Maccheroni, and Marinaci (2004) and Klibanoff, Marinacci, and Mukerji (2005) provide alternative approaches to separate ambiguity and the decision maker's attitude toward ambiguity. That is, they know the true mean and variance of the durability of their products. Like producers, sophisticated consumers do not face ambiguity about product quality, but unsophisticated consumers face ambiguity about product quality.

The researcher constructs consumers' beliefs in product quality. Let the random variable $\tilde{X}_j$ represent the durability of producer $j's$ product, whose true mean and true variance are denoted by m_j and v_j respectively. Thus, a sophisticated consumer's beliefs in this random variable are given by $m_j^s = m_j,\ v_j^s = v_j.$ In contrast, an unsophisticated consumer believes that the mean of the random variable $\tilde{X}_j$ takes value $m_j^u \in \left(m_{j\min}^u,...,m_{j\max}^u\right)$ and the variance takes value $v_j^u \in \left(v_{j\min}^u,...,v_{j\max}^u\right).$ The consumer believes that all permissible pairs of the means and variances are possible. Again, the researcher assumes the true mean and the true variance are convex combinations of the extreme values of the means and variances, respectively.

This random variable has mean equal to $E\left[\tilde{X}_j\right] = \frac{1}{\lambda_j}$ and variance equal to $var\left[\tilde{X}_j\right] = \frac{1}{\left(\lambda_j\right)^2}.$ For completeness provide their derivations. Using the probability density specified above and applying L'Hospital's rule, we get

$$E\left[\tilde{X}_j\right] = \int_0^\infty \lambda_j t e^{-\lambda_j t} dt = ... = \frac{1}{\lambda_j}$$

$$E\left[\left(\tilde{X}_j\right)^2\right] = \int_0^\infty \lambda_j t^2 e^{-\lambda_j t} dt = ... = \frac{2}{\left(\lambda_j\right)^2}$$

$$var\left(\tilde{X}_j\right) = E\left[\left(\tilde{X}_j\right)^2\right] - \left(E\left[\tilde{X}_j\right]\right)^2 = \frac{1}{\left(\lambda_j\right)^2}$$

Mathematically, we express the true parameters as

$m_j = b_j m_{j\min}^u + (1-b_j) m_{j\max}^u,\ v_j = h_j v_{j\min}^u + (1-h_j) v_{j\max}^u$, where $b_j, h_j \in [0,1]$.

The summary of consumer $i's$ beliefs about producer $j's$ product by

θ_j^i, for $i = s,u$ and $j = 1,...,N.$ (1)

3.4. *Demand Function* for *a Product*

The market demand for producer $j's$ product is formulated. Let the random variable $\tilde{Y}_j$ denote quality (durability) of this producer's product. It is assume that this random variable is exponentially distributed with parameter $\lambda_j = \frac{1}{m_j}$ where m_j and v_j denote the true mean and true variance of the random variable, respectively.

The mean represents the expected elapsed time until the product breaks down for the first time. For simplicity, the researcher assumes that these random variables $\tilde{Y}_j$ are mutually independent of $j = 1,...,N.$

Let $\eta_j = \eta_j(m_j, T_j)$ denote expected durability of the producer's product with a warranty, in which T_j is the duration of the product warranty. Product warranties are influenced by market participants' beliefs in product quality. The producer chooses T_j and consumers observe it. Let $t_j = t_j(m_j, T_j, q_j)$, where q_{ji} is the quantity of the product produced by producer j, denote the total expected number of replacements performed by the producer or designated agent. In the case of an automobile industry, this is a dealership. The mean of the product's durability determines the expected number of breakdowns for a single product that occur within the time period of the warranty. The entire quantity of the producer's output is purchased by consumers because the product market clears. Thus, the producer's output has an impact on the total expected number of repairs performed by the producer. The record of the expected durability of a product with a warranty and the total expected number of repairs performed within the time period of the warranty in the lemma below. These results will be helpful in stating the market participants' choice problems.

Lemma 1: The expected durability η_j with a warranty for producer $j's$ product and the expected number t_j of replacements performed by the producer are given by the equations

$$\eta_j = m_j + T_j \quad (2)$$

$$t_j = \frac{T_j}{m_j} q_j \quad (3)$$

The approach taken in the literature is that consumers derive satisfaction from the quality of the products they purchase. Consumers value the level of satisfaction in terms of money.[9] Because the measure of the quality of a product by the durability of the product, a highly durable product provides consumers with a high level of satisfaction. From equation (2) in Lemma 1 it is noted that a consumer's budget constraint is of this form:

$$\tilde{W} = a + (\tilde{V} + T - p)x \quad (4)$$

The parameter a is a nonnegative constant, which represents money in the consumer's bank account that the consumer now consumes. The variable $\tilde{V}$ represents the monetary value of the product durability. The choice variable T is the monetary value of the duration of the product warranty. The researcher denotes the price of the product by p which clears the market for the product. The choice variable x denotes the quantity of the product that the consumer purchases.

Consumers have mean-variance utility functions of this form:

$$E[u(\tilde{W})] = a + (m_j + T - p)x - 0.5\bar{\gamma}vx^2 \quad (5)$$

The first two terms in (5) follow from the consumer's budget constraint (4), in which m_j is the monetary value of the mean durability for producer *j*. The third term is intuitively clear.

If the durability of a product varies, a consumer derives disutility. The parameter $\gamma > 0$ denotes the coefficient of absolute risk aversion. For notational convenience, the researcher assumes that all consumers have the same level of risk aversion.

[9] See, for example, Heal (1977).

The researcher has solved the consumer's choice problem. In addition, the researcher has used the market clearing condition for a product to establish the market demand function for the product. The idea is that a producer maximizes profits using the same approach as a monopolist does, so he needs the market demand function for his product.

Problem 1: Consumer i solves this problem:

$$U^i = \max_{(x^i)} \min_{(\theta^i)} a^i + \sum_{j=1}^{n} \left(m_j^i + T_j - p_j \right) x_j^i - \frac{1}{2} \gamma v_j^i \left(x_j^i \right)^2 \tag{6}$$

The minimum value of the expected utility function occurs if $m_j^s = m_j,\ v_j^s = v_j$ for sophisticated investors and $m_j^u = m_{j\min}^u,\ v_j^u = v_{j\max}^u$ for unsophisticated investors, in which m_j and v_j are the monetary values of the mean and variance of product durability for producer j and $T_j$is the monetary value of the duration of product warranty for producer j. Then the usage of the standard techniques in microeconomics was used to solve the utility maximization part of Problem 1. By taking the derivative of the expected utility function in (6) with respect to the choice variable x_j^i and setting the derivative to zero yields the demand functions for $i = s, u$:

$$x_j^s = \frac{m_j + T_j - p_j}{\bar{\gamma} v_j} \tag{7}$$

$$x_j^u = \begin{cases} \dfrac{m_{j\min}^u + T_j - p_j}{\bar{\gamma} v_{j\max}^i} \text{ if } p_j < m_{j\min}^u + T_j \\ 0 \text{ if } m_{j\min}^u + T_j \le p_j \end{cases} \tag{8}$$

If the value of the warranty for a product is too small to support the price of the product, the consumer does not buy the product. A consumer's non-participation in a product market is an outcome of rational behavior. To induce consumers to participate in their product market, producers use two mechanisms to prevent their product market from breaking down. Producers can improve product quality.

Producers can increase the duration of product warranty. To improve his sales, a producer chooses the warranty to induce consumer participation in the market for his product.

The market clearing condition for producer $j's$ product is given by the relation

$$\alpha^s x_j^s + \alpha^u x_j^u = q_j \tag{9}$$

The substitution of the consumers' demand functions for the product into the product market clearing condition to obtain the market demand function for the product. As expected, the product demand function is downward sloping.

Lemma 2: The market demand function and consumer $i's$ demand function for producer $j's$ product are given by:

$$p_j = w_j^s \left(m_j + T_j \right) + w_j^u \left(m_{j\min}^u + T_j \right) - \bar{\gamma} a_j q_j \tag{10}$$

$$w_j^s = \frac{\alpha^s v_{j\max}^i}{\alpha^s v_{j\max}^i + \alpha^u v_j},\ w_j^u = \frac{\alpha^u v_j}{\alpha^s v_{j\max}^i + \alpha^u v_j}, a_j = \frac{v_j v_{j\max}^i}{\alpha^s v_{j\max}^i + \alpha^u v_j}$$

The market demand function (10) yields a downward sloping curve in (q_j, p_j) space. The demand for the producer's product is affected by the market participants' beliefs in the quality of the product. The product warranty offered by the producer also affects demand for the product. This will be useful in determining producer $j's$ output and the price of the output.

Each producer is now equipped with a market demand function for this product. The producer's production functions for his product and replacement parts. To simplify things, it has assumed separate production plants.[10]

3.5Producer Profit Maximization Problem

Producers use decreasing returns to scale technology to produce their products. The total cost is represented as the production for producer j by a convex cost function of the form:

$$TC_j^p = c_1\left(q_j\right)^2 \tag{11}$$

The choice variable is the quantity q_j of output that the producer produces. Each producer faces the same cost function regardless of the quality of the product. The idea is that even if producers have access to the same technology, their management teams have different quality control mechanisms.

It is assumed producers use decreasing returns to scale to produce replacement parts to support their warranties. We represent the total cost of producing replacement parts which is given by a convex cost function of the form:

$$TC_j^m = c_2\left(t_j\right)^2 \tag{12}$$

The variable t_j is the total expected number of repairs done by the producer.

[10]I follow the approach of Gal-Or (1989)

To reduce the cost of production, producers use scraps of broken items to produce the parts. Thus, the unit costs satisfy the relation $c_2 \le c_1$.Thus, the total cost function is given by:

$$TC_j = TC_j^p + TC_j^m = c_1\left(q_j\right)^2 + c_2\left(t_j\right)^2 \tag{13}$$

The expected profit for producer j is given by the equation:

$$E\left[\pi_j\right] = p_j q_j - TC_j \tag{14}$$

The first term is total revenue, and the second term is total cost.

By substituting the market demand function specified in (10), the total number t_j of repairs specified in (2) and the total cost function (13) into the expected profit function (14), the researcher obtain producer $j's$ profit maximization problem below.

Problem 2: Producer j chooses the quantity q_j of output and the value T_j of warranty to support the producer's product in order to maximize expected profit. In other words, the producer solves the problem:

$$\pi_j = \max_{q_j, T_j}\left(w_j^s\left(m_j + T_j\right) + w_j^u\left(m_{j\min}^u + T_j\right) - \gamma a_j q_j\right)q_j$$

$$-c_1\left(q_j\right)^2 - c_2\left(T_j\right)^2\left(m_j\right)^{-1}\left(q_j\right)^2\Big) \tag{15}$$

The record of the closed-form representations of the equilibrium quantities in the theorem below.

Theorem The equilibrium variables for producer j and consumer i are given by the relations:

$$T_j = f\left(c_1, c_2, m_j w_j^s, w_j^u, m_j^u \min\right) \tag{16}$$

$$p_j = w_j^s\left(m_j + T_j\right) + w_j^u\left(m_{j\min}^u + T_j\right) - \gamma a_j q_j \tag{17}$$

$$q_j = \frac{w_j^s\left(m_j + T_j\right) + w_j^u\left(m_{j\min}^u + T_j\right)}{2\left(c_1 + c_2\left(T_j\right)^2\left(m_j\right)^{-1}\right)} \tag{18}$$

This is a brief description of the implications of the theorem for pricing and the product warranty. First, the market price of the product depends on the beliefs of the market participants. The price increases if the market participants view the product to be of higher quality. The price increases if the producer offers a product warranty of longer duration, suggesting that consumers pay more for a higher product warranty. The pricing of product warranties depends on the beliefs of market participants about product quality. Specifically, the warranty of a product is positively related to the manufacturer's belief in the quality of his product and negatively related to consumers' beliefs in the quality of the product. If consumers are pessimistic about the quality of a product, they pay more to get a long duration of the product warranty. The unit cost of producing replacement items is negatively related to the duration of product warranty. Indeed, a producer will support his product with a long duration warranty if the cost of producing replacement parts is low. A producer's output is positively related to consumers' belief in product quality. If consumers are pessimistic about the quality of the producer's product, their demand for the product is low. The implication is that producers that want to remain competitive in the industry must improve the quality of their products. The effect of competition on warranties is that producers provide consumers with high quality product, leading to long duration warranties. A benchmark for the study is the case in which producers and consumers have homogeneous beliefs in product durability. When consumers and producers have homogeneous beliefs in product quality, then the equilibrium variables for producer i and consumer j are given by the relations:

$$T_j = \frac{\left(m_j\right)^2}{2c_2 q_j} \tag{19}$$

$$p_j = m_j + T_j - \gamma b_j a_j q_j \tag{20}$$

$$q_j = \frac{\left(m_j + T_j\right)}{2\left(c_1 + c_2\left(T_j\right)^2\left(m_j\right)^{-1}\right)} \tag{21}$$

If participants have homogeneous beliefs in product quality, then consumers pay high premiums for high quality products and low premiums for low quality products. The results are influenced by variability of product quality as before. Consumers purchase the same quantity of a producer's product. If consumers' degree of risk aversion is allow differing, and then they purchase different quantities of the product. The net gain to the producer depends on the producer's belief in the quality of the product and the cost of the warranty. It is positively related to the producer's belief in product quality and negatively related to the unit cost of producing parts for repairing consumers' products. More precisely, this is given by the relation:

$$\left(T_j q_j - c_2 \frac{T_j^2 q_j^2}{m_j^2}\right) = \frac{m_j^2}{4c_2} \tag{22}$$

Producers derive positive net benefits from providing consumers with insurance for their products. This suggests product warranties are on average profitable as the insurance industry suggests. Another implication is that the parts industry is a big business, especially for repairs that are not covered by warranties.

4. An overview of the Effect of Consumers' Beliefs on Product Price

4.1 Effect of Consumers' Beliefs on Product Price

The role of price in consumer behavior is certainly complex. It is argued that consumers' beliefs on product quality affect product price. Economists have long observed that, in imperfect information markets consumers will use prices to signal level of quality among unfamiliar alternatives. In marketing, a number of researchers have confirmed that consumers frequently do subscribe to a price is a reliable indicator of product quality belief (Monroe 1976; Olson 1977).

4.2 Effect of Consumers' Beliefs on Product Warranty

It is argued that consumer utility is affected by warranty duration as a signal of product quality. As expected, longer warranty increases consumers' utility and consumers are more likely to buy products with longer warranties. There are two reasons consumers may prefer longer warranty duration. First, consumers may consider warranty as a signal of quality because they know warranties are costly to offer by firms. In the case of durable goods where consumers do not have full information on quality, consumers may rely on the offer of warranties to make informed decisions. Second, consumers might prefer to buy a product with a longer warranty because they are risk averse. In this case, a warranty is insurance against product failure. With an assumption that consumers are heterogeneous in risk aversion, more risk averse consumers prefer a longer warranty as it reduces the risk of product failure. Choi and Ishii (2011) seek empirical evidence for the role of warranties as signals of unobservable quality. They adapt the linear random utility model of consumer automobile demand to investigate the extent to which warranties affect consumer choice and the extent to which this estimated warranty effect is due to risk aversion and signaling motives. The theory implies a positive relationship between product quality and warranty duration because only highly-quality firms can afford long warranties because of their associated costs of fulfillment. On the other hand, the firms' signal product quality to consumers through warranties in the presence of consumer moral hazard and motivate the firm to invest in product quality and supply high-quality products. Many empirical studies have shown that warranties have a significant impact on consumer product choice and that they would pay a premium for products with better warranty terms.

5. Conclusion

In addition to mechanisms like reputation, advertising and product-specific investment to market products to consumers, producers use warranties for a number of purposes. I argue that product warranties are a market mechanism that plays a role similar to the lemon's law to prevent the collapse product markets. Specifically, product warranties induce ambiguity-avers consumers to participate in product markets. In pricing their product warranties, producers consider consumers' beliefs and their own beliefs about the quality of their products.

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Appendix A: Proofs

A

.1. *Proof of Lemma 1*

It is consider a general producer. Let denote m_j the mean of this producer $j's$ product. Let $\tilde{X}_j$ denote the exponential random variable that represents the durability of the product, that is, $\tilde{X}_j$ has parameter $\lambda_j = \frac{1}{m_j}$.

Thus, we get

$$E\left[\tilde{X}_j + T_j\right] = m_j + T_j.$$

(A1)

Let $\tilde{Y}_1,...,\tilde{Y}_r$ denote exponentially distributed random variables representing product breakdowns within the duration of the warranty for the product. The random variable $\tilde{Y}_1$ is the waiting time until the product breaks down for the first time. The random variable $\tilde{Y}_2$ is the waiting time between the first breakdown and the second breakdown. The other waiting times are defined similarly.

For simplicity, I assumed that these waiting times are equally spaced, that is,

$$E\left[\tilde{Y}_1\right] = ... = E\left[\tilde{Y}_r\right] = m_j.$$

Let the random variable $Y_j = \tilde{Y}_1 + ... + \tilde{Y}_{r_j}$ represent the total waiting time within the duration, T_j, of product warranty. Because an exponential random variable has no memory, we have $E\left[\tilde{Y}\right] = E\left[\tilde{Y}_1\right] + ... + E\left[\tilde{Y}_r\right] = r_j m_j$. Thus, the total expected number of product breakdowns is given by the equation:

$$r_j = \frac{T_j}{m_j}.$$

Let q_j denote the producer's amount of output. This implies that the total expected number t_j of repairs performed within the duration of the product warranty is given by

$$t_j = \frac{T_j}{m_j} q_j.$$