

## The Mental Logic Theory and De Morgan's Laws

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### Abstract

*Macbeth et al. carry out an experiment on DeMorgan's laws and interpret that their results support the mental models theory and undermine formal rules theories such as that of Rips. In this paper, I try to show that, although Macbeth et al.'s results are clearly consistent with the mental models theory and enable to question Rips' approach, they are not incoherent with other formal or syntactic frameworks; in particular, they are not inconsistent with the mental logic theory. Thus, I comment some aspects of this last theory that are often misinterpreted and misunderstood.*

**Keywords:** DeMorgan's laws, formal rules, mental logic, mental models, reasoning

### 1. Introduction

There are several theories that explain human reasoning. However, in this paper, I will mainly focus on only two of them: the mental models theory (from now on, MM) and the mental logic theory (from now on, ML). MM (e.g., Byrne & Johnson-Laird, 2009; Johnson-Laird, 1983, 2010, 2012; Johnson-Laird, Byrne, & Girotto, 2009; Khemlani & Johnson-Laird, 2009; Khemlani, Orenes, & Johnson-Laird, 2012; Oakhill & Garnham, 1996; Orenes & Johnson-Laird, 2012) is a theory that, undoubtedly, is being very successful, since it is proving that it can explain and predict reasoners' answers in most experimental reasoning tasks. In fact, in some papers, its proponents try to demonstrate that MM can predict cognitive phenomena that other theories cannot even explain. This is the case, for example, of Orenes and Johnson-Laird's (2012) research, in which they seem to claim that only MM can explain and predict their participants' behavior in reasoning problems with different versions of certain 'paradoxical inferences' related to conditional and disjunction.

In this context, a very interesting work is that of Macbeth et al. (2014). In that work, an experiment is carried out with the goal of checking how people interpret and understand DeMorgan's (1847) laws. However, the most relevant aspect of Macbeth et al.'s (2014) research to this paper is that they state that their results clearly support MM and are incompatible with the predictions of the formal rules theories such as that of Rips (1994, 2011).

In my view, it is obvious that the results achieved by Macbeth et al. (2014) are consistent with MM and can lead one to think that Rips' (1994, 2011) framework must be rejected. Nevertheless, I think that such results are not incompatible with the predictions of all the formal theories, in particular, with those of ML. This last theory, which is exposed, described, and commented in details, for example, in Braine and O'Brien (1998a), is not equivalent to standard propositional calculus or frameworks such as that of Gentzen (1934). It is a theory based on formal rules, but only on the formal rules that, according to empirical evidence, individuals really use. In this way, as I will show below, DeMorgan's laws do not have the same status in ML as in standard propositional calculus, and, for this reason, Macbeth et al.'s (2014) results cannot be related to the predictions of ML.

To prove it, I will begin by explaining how MM understand conjunctions and disjunctions. Secondly, I will expose which the predictions of MM corresponding to DeMorgan's laws are and why Macbeth et al.'s (2014) experiment confirms such predictions. Finally, I will show that Macbeth et al.'s (2014) results do not concern the theses of ML.

### 2. Conjunctions, Disjunctions, and MM

MM states that reasoners make inferences by considering the possibilities that correspond to both the premises and the conclusion.

Thus, they tend to identify the possibilities in which both the premises and the conclusion are true. MM assumes that classical logical operators have both ‘mental models’ (which are recovered without effort) and ‘fully explicit models’ (which need certain effort to be recovered). However, for the purposes of this paper, it is only relevant to pay attention to the fully explicit models of conjunction and inclusive disjunction. Such fully explicit models are as follows:

Conjunction (p and q):

$p - q$

Inclusive disjunction (p or q, or both):

$p - q$

$p - \neg q$

$\neg p - q$

Where ‘ $\neg$ ’ stands for denial.

As it can be noted, these fully explicit models indicate that conjunction can only be true in one case: when both p and q are true. Nevertheless, inclusive disjunction can be true in three cases: when both p and q are true, when p is true and q is false, and when p is false and q is true. The problem of DeMorgan’s laws is that, in such laws, conjunction and inclusive disjunction are denied. As it is well known, DeMorgan’s laws are the following:

Law 1 (from now on, L1):  $\neg(p \wedge q) = \neg p \vee \neg q$

Law 2 (from now on, L2):  $\neg(p \vee q) = \neg p \wedge \neg q$

Where ‘ $\wedge$ ’ is conjunction and ‘ $\vee$ ’ represents inclusive disjunction.

Khemlani et al. (2012) proposed a way to identify the models corresponding to denied propositions. Thus, according to them, the models of a denied proposition are the complement of that same proposition when it is not denied. The models of  $\neg(p \wedge q)$  are hence:

$p - \neg q$

$\neg p - q$

$\neg p - \neg q$

And the only model of  $\neg(p \vee q)$  is this one:

$\neg p - \neg q$

It is interesting to see that, based on this account, it can be said that MM proves that L1 and L2 are correct laws. This is evident if it is considered that the models of  $\neg p \vee \neg q$  are the same as those of  $\neg(p \wedge q)$ , and that the models of  $\neg p \wedge \neg q$  are the same as those of  $\neg(p \vee q)$ . Indeed, the models corresponding to  $\neg p \vee \neg q$  are obviously these ones:

$\neg p - \neg q$

$\neg p - q$

$p - \neg q$

On the other hand,  $\neg p \wedge \neg q$  only refers to this model:

$\neg p - \neg q$

But the most important aspect of this account is that it has a relevant consequence: if  $\neg(p \wedge q)$  is linked to three models and  $\neg(p \vee q)$  is only related to one model, L1 must be a harder law than L2 for reasoners. Macbeth et al. (2014) pay attention to this prediction and carry out an experiment in order to check it.

### **3. Is Really L1 More Difficult than L2?**

Basically, Macbeth et al.’s (2014) experiment consists of experimental conditions in which a proposition with the formal structure of L1 or L2 is presented in capital letters and it is followed by four propositions in small letters that refer to different possible interpretations of the proposition in capital letters. Evidently, participants’ task is to indicate what proposition in small letters expresses the exact meaning of the proposition in capital letters. This is an example provided by them related to a denied conjunction:

“IT IS NOT TRUE THAT: LONDON IS A CITY AND AFRICA IS A CONTINENT

- a. London is not a city and Africa is not a continent.
- b. London is not a city or Africa is not a continent.\*
- c. If London is not a city, then Africa is not a continent.
- d. Either London is not a city or else Africa is not a continent, but both things cannot be right at the same time” (Macbeth et al., 2014, p. 141).

Of course, ‘\*’ in the option b means that that is the correct option, but what is important here is that Macbeth et al. (2014) checked that, certainly, L1 was harder than L2 for their participants. Indeed, they tended to choose the right option more often when the proposition in capital letters corresponded to  $\neg(p \vee q)$ , and less frequently when it had the structure  $\neg(p \wedge q)$ . Macbeth et al. (2014) interpreted these results as a proof that the predictions of MM on DeMorgan’s laws are correct. In their view, to understand the real meaning of L1 requires, as said, three possibilities to be considered. On the contrary, the actual meaning of L2 can be captured detecting only one model.

Nonetheless, Macbeth et al. (2014) think that other relevant consequence of their results is that they show that other alternative theories do not hold. In this way, they mention that formal theories such as that of Rips and ML propose predictions different for the tasks included in their experiment. However, they only focus on Rips’ theory and only explain why their results are inconsistent with this theory. In my opinion, their account of Rips’ theory is right. It requires, as they comment, three inferential steps for L1 and five for L2, which means that, according to it, L1 should be easier than L2. Given that the explanation offered by Macbeth et al. (2014) is clear enough and convincing, I think that it is not necessary to reproduce it here again. Maybe it is better to pay attention to ML, since, as said, Macbeth et al. (2014) do not analyze it. I think that to do so is important because the proponents of other theories often tend to misunderstand and misinterpret ML, which tend to be identified with Rips’ framework or standard propositional calculus. As I will explain in the next section, it is a mistake.

#### 4. ML and DeMorgan’s Laws

Indeed, it could be thought that an incorrect aspect of Macbeth et al.’s (2014) paper is precisely that it appears to assume that Rips’ theory and ML are equivalent. I am saying this because one of its paragraphs (Macbeth et al., 2014, p. 139) begins by mentioning both Rips’ approach and ML, but, after that, only Rips’ predictions are commented. Maybe this is not the case but, because that mistake, as mentioned, is frequently made, one might ask why Macbeth et al. (2014) quote ML along with Rips’ theory and then they only review Rips’ approach.

Be that as it may, it seems obvious that what is necessary now is to indicate which the real predictions of ML on DeMorgan’s laws are. However, to do that is very easy, since ML has no predictions on L1 and L2. As said, ML does not accept all the formal rules of standard propositional logic, but only those rules that, following empirical evidence, people truly use in a natural way. ML distinguishes between different kinds of rules (‘core’ rules, ‘feeder’ rules, ‘incompatibility’ rules...) and consists of a program that establishes the order in which such rules are often applied. Nonetheless, two points regarding ML are important here: at least in the description presented by Braine and O’Brien (1998b), ML has no rules corresponding to  $\neg(p \vee q)$  and has only one rule corresponding to  $\neg(p \wedge q)$ . This last rule is the core schema 4 in Braine and O’Brien (1998b), is based on Chrysippus’ modus ponendo tollens, and has a structure similar to this one:

$$\neg(\alpha_1 \wedge \dots \wedge \alpha_n), \alpha_i; \text{ ergo } \neg(\alpha_1 \wedge \dots \wedge \alpha_{i-1} \wedge \alpha_{i+1} \wedge \dots \wedge \alpha_n)$$

There is no rule in ML that enables to draw  $\neg p \vee \neg q$  from  $\neg(p \wedge q)$  or  $\neg p \wedge \neg q$  from  $\neg(p \vee q)$ . Therefore, it is very hard to know which the predictions of ML to which Macbeth et al. (2014) refer are.

In my view, it is an actual problem the fact that ML continues to be a very unknown theory, since, in this particular case, it can lead the reader to assume the idea that ML raises predictions on DeMorgan’s laws different to those of MM. Nevertheless, what really happens is that, as indicated, ML does not have particular predictions on such laws. Based on ML, given that they are not basic rules that people daily use, it can be said that it is possible that some reasoners can identify the equivalences between  $\neg(p \wedge q)$  and  $\neg p \vee \neg q$ , and between  $\neg(p \vee q)$  and  $\neg p \wedge \neg q$ . Nonetheless, this is not guaranteed. Macbeth et al.’s (2014) experiment hence does not cause difficulties to ML, since their results are, in general, compatible with the main theses of ML.

But let us suppose that we assume ML. In that scenario, what would Macbeth et al.’s (2014) results mean? This question could be answered by considering Macbeth et al.’s research not to be a study on human reasoning, but on human ability to process expressions in natural language.

From this point of view, the results obtained by Macbeth et al. (2014) would only reveal how human beings understand certain information, not how they reason about that information, and, therefore, would not have an influence on the fundamental theses of ML. In particular, such results would only show that it is easier for people to understand the real meaning of  $\neg(p \vee q)$  than that of  $\neg(p \wedge q)$ .

## 5. Conclusions

Undoubtedly, MM is a powerful theory that can explain and predict most reasoners' behaviors in experimental tasks. This paper has not been intended to demonstrate the opposite. My aim has only been to show that the experimental results that, according to the literature on cognitive science, support MM do not always necessarily prove that other theories are false. In this way, it can be stated that, as it can be checked paying attention to the previous arguments, Macbeth et al.'s (2014) research does not really have an influence on ML.

The main problem is that, as commented, ML is a misunderstood and unknown theory. It is often considered to be a theory akin to that of Rips (1994) or to standard propositional calculus. However, such assumptions are, as also indicated, a great mistake, since ML is very different to those frameworks in many aspects. This is the reason why several experimental results that appear to show that ML is not a valid theory do not actually do so. In most cases, the experimental results only reveal that approaches such as that of Rips or those based on classical logic have certain problems that must be solved, but not that ML has such problems too.

On the other hand, I think that this discussion is not adequately raised. Researches often try to prove that a particular theory is correct and all the other theories are incorrect, or that a particular experimental result can only be explained or predicted by one theory, and not by all the other theories. This perspective can be erroneous because, as claimed by one of the proponents of ML, this last theory must not be exclusive (O'Brien, 1998). In one sense or in some manner, MM and ML can be right at the same time, since human reasoning can work resorting, in some cases, to mental models and, in other cases, to formal schemata. It is clear that this thesis is controversial and problematic, but there are already some studies that try to find links between semantic mental models and syntactic rules, or that show how both semantic and syntactic approaches can account for certain phenomena (e.g., López-Astorga, 2013, 2014a, 2014b), which means that it can be worth researching in this direction.

In any case, as mentioned, Macbeth et al.'s (2014) study is interesting even although ML is assumed, since it can be thought that their results explain how individuals tend to interpret certain expressions. In addition, other suggestive possibility could be to propose an extension of ML including L2. Because it is relatively less difficult for people to understand the fact that  $\neg(p \vee q)$  means  $\neg p \wedge \neg q$  than the fact that  $\neg(p \wedge q)$  means  $\neg p \vee \neg q$ , it could be considered to add the following rule to the set of schemata of ML:

$\neg(\alpha \vee \beta)$ ; ergo  $\neg\alpha \wedge \neg\beta$

Nevertheless, to do this, further empirical research would be needed. Firstly, it would be necessary to determine whether or not the data are significant enough to adopt this new rule. On the other hand, if that rule was admitted, other important point would have to be decided, since what type of schema (core, feeder...) the rule is would have to be indicated. Finally, the limitations of Macbeth et al.'s (2014) study would have to be taken into account as well. Such limitations are, for example, the use of only counterfactuals or of only concrete texts, and not abstract texts.

Macbeth et al. (2014) acknowledge that their research has that sort of limitations, but, in my view, the most important aspect of their experiment is that it does not prove that all the formal or syntactic theories are false. Thus, regardless of the possibility of adopting this new rule, it can be said that ML is not inconsistent with the results achieved by Macbeth et al. (2014).

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## 6. References

- Braine, M. D. S. & O'Brien, D. P. (Eds.) (1998a). *Mental Logic*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Braine, M. D. S. & O'Brien, D. P. (1998b). The theory of mental-propositional logic: Description and illustration. In M. D. S. Braine & D. P. O'Brien (Eds.), *Mental Logic* (pp. 79-89). Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Byrne, R. M. J. & Johnson-Laird, P. N. (2009). "If" and the problems of conditional reasoning. *Trends in Cognitive Science*, 13, 282-287.
- DeMorgan, A. (1847). *Formal Logic: Or, the Calculus of Inference Necessary and Probable*. London, UK: Taylor & Walton.
- Gentzen, G. (1934). Untersuchungen über das logische. I. *Mathematische Zeitschrift*, 39, 176-210.
- Johnson-Laird, P. N. (1983). *Mental Models. Towards a Cognitive Science on Language, Inference, and Consciousness*. Cambridge, MA: Harvard University Press.
- Johnson-Laird, P. N. (2010). Against logical form. *Psychologica Belgica*, 5(3&4), 193-221.
- Johnson-Laird, P. N. (2012). Inference with mental models. In K. J. Holyoak & R. G. Morrison (Eds.), *The Oxford Handbook of Thinking and Reasoning* (pp. 134-145). New York, NY: Oxford University Press.
- Johnson-Laird, P. N., Byrne, R. M. J., & Girotto, V. (2009). The mental models theory of conditionals: A reply to Guy Politzer. *Topoi*, 28, 78-80.
- Khemlani, S. & Johnson-Laird, P. N. (2009). Disjunctive illusory inferences and how to eliminate them. *Memory & Cognition*, 37(5), 615-623.
- Khemlani, S., Orenes, I., & Johnson-Laird, P. N. (2012). Negation: A theory of its meaning, representation, and use. *Journal of Cognitive Psychology*, 24(5), 541-559.
- López-Astorga, M. (2013). Are conditional and disjunction really comparable? *Universum*, 28(2), 229-242.
- López-Astorga, M. (2014a). Mental models and syntactic rules: A study of the relations between semantics and syntax in inferential processes. *Annals of the University of Craiova, Philosophy Series*, 33(1), 107-117.
- López-Astorga, M. (2014b). Conditional non-contradiction, mental models, and syntactic rules. *Pensée*, 76(8), 85-91.
- Macbeth, G., Razumiejczyk, E., Crivello, M. C., Bolzán, C., Pereyra Girardi, C. I., & Campitelli, G. (2014). Mental models for the negation of conjunctions and disjunctions. *Europe's Journal of Psychology*, 10(1), 135-149.
- Oakhill, J. & Garnham, A. (Eds.) (1996). *Mental Models in Cognitive Science. Essays in Honour of Phil Johnson-Laird*. Hove, UK: Psychology Press.
- O'Brien, D. P. (1998). Mental logic and irrationality: We can put a man on the moon so why can't we solve those logical reasoning problems? In M. D. S. Braine & D. P. O'Brien (Eds.), *Mental Logic* (pp. 23-43). Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers.
- Orenes, I. & Johnson-Laird, P. N. (2012). Logic, models, and paradoxical inferences. *Mind & Language*, 27(4), 357-377.
- Rips, L. J. (1994). *The Psychology of Proof: Deductive Reasoning in Human Thinking*. Cambridge, MA: Massachusetts Institute of Technology (MIT) Press.
- Rips, L. J. (2011). *Lines of Thought: Central Concepts in Cognitive Psychology*. New York, NY: Oxford University Press.